Introduction to Three-phase Circuits

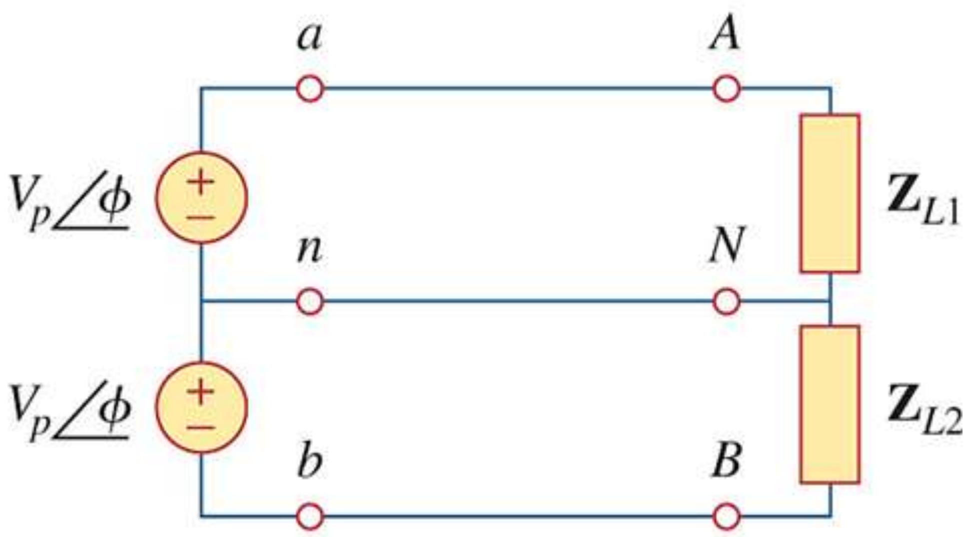
Balanced 3-phase systems

Unbalanced 3-phase systems

Introduction to 3-phase systems

Single-phase two-wire system:

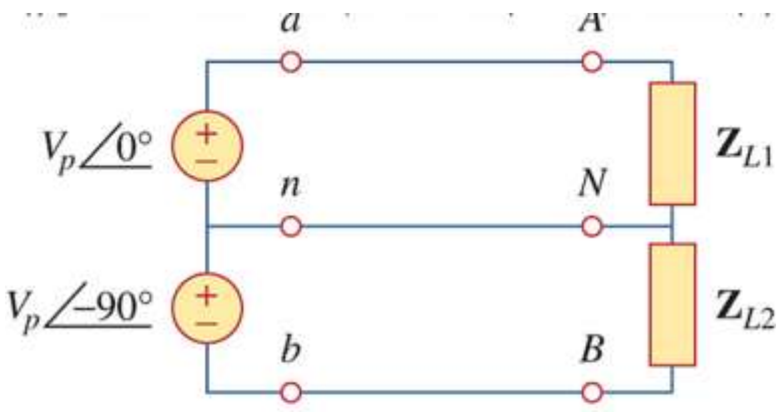
- Single source connected to a load using two-wire system



Single-phase three-wire system:

- Two sources connected to two loads using three-wire system
- Sources have EQUAL magnitude and are IN PHASE

Circuit or system in which AC sources operate at the same frequency but different phases are known as polyphase.

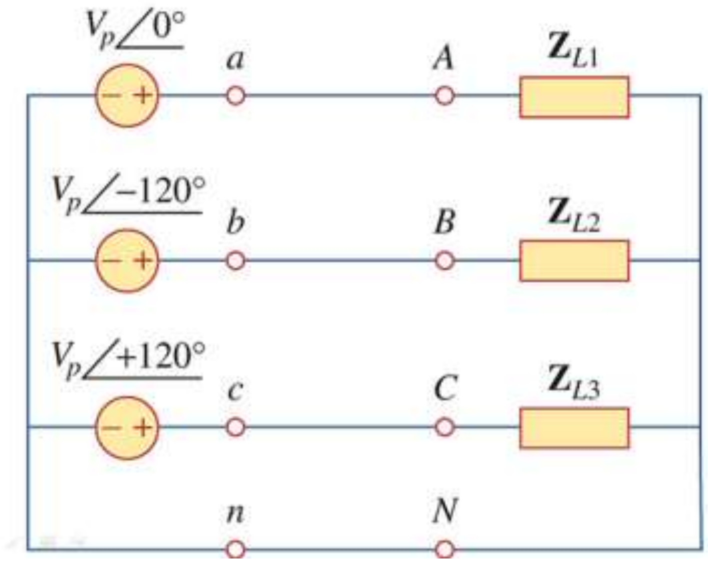


Balanced Two-phase three-wire system:

- Two sources connected to two loads using three-wire system
- Sources have EQUAL frequency but DIFFFERENT phases

Two Phase System:

- A generator consists of <u>two coils</u> placed perpendicular to each other
- The voltage generated by one lags the other by 90°.



Balanced Three-phase four-wire system:

- Three sources connected to 3 loads using four-wire system
- Sources have EQUAL frequency but DIFFFERENT phases

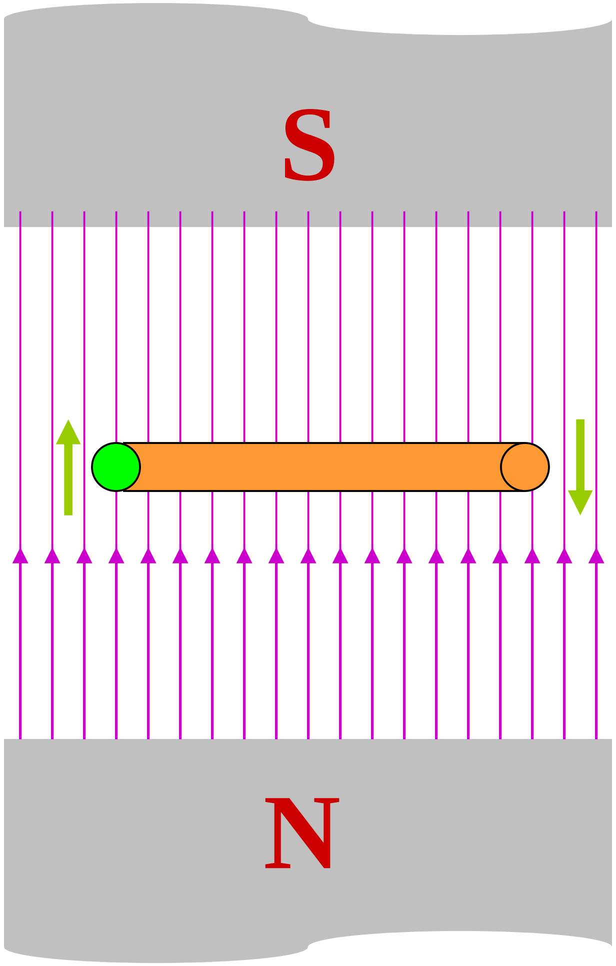
Three Phase System:

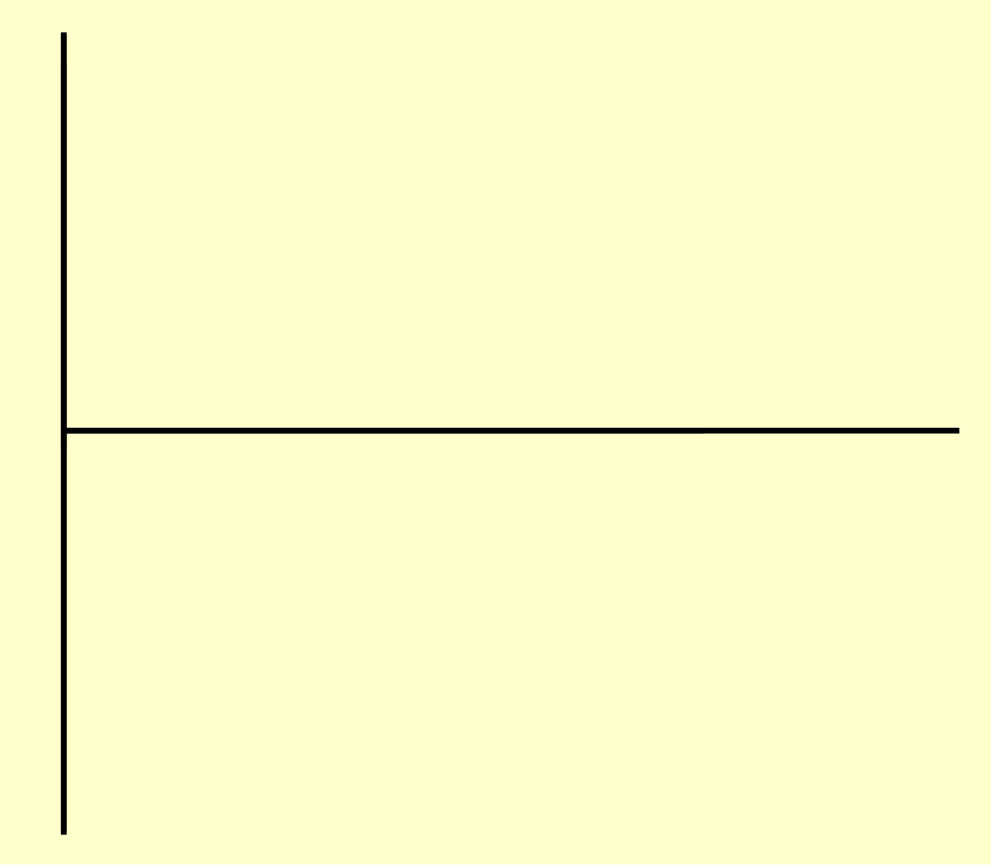
- A generator consists of <u>three coils</u> placed 120° apart.
- The voltage generated are equal in magnitude but, out of phase by 120°.
- Three phase is the most economical polyphase system.

AC Generation

- Three things must be present in order to produce electrical current:
  a) Magnetic field
  b) Conductor
  c) Relative motion
- Conductor cuts lines of magnetic flux, a voltage is induced in the conductor
- Direction and Speed are important

GENERATING A SINGLE PHASE

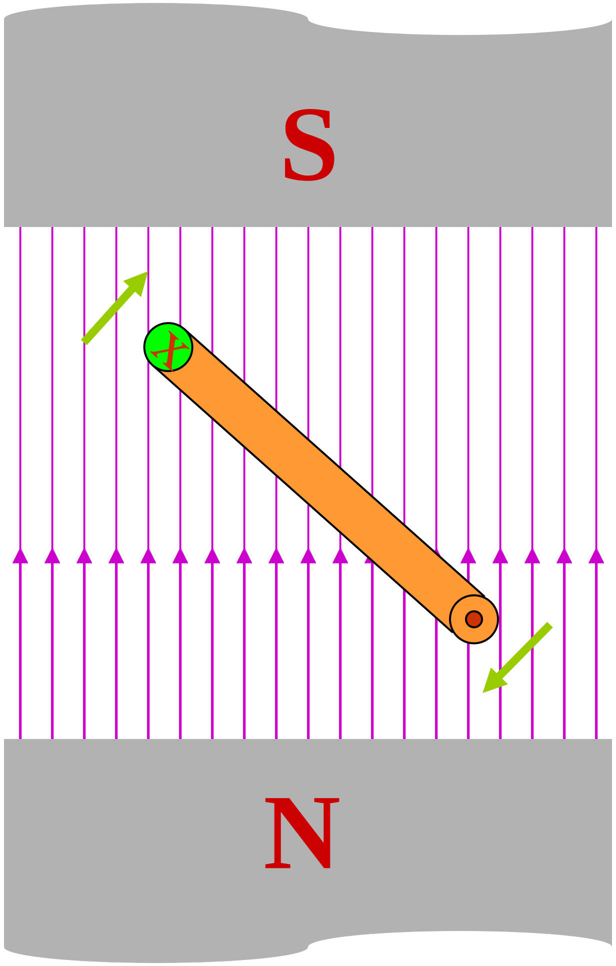


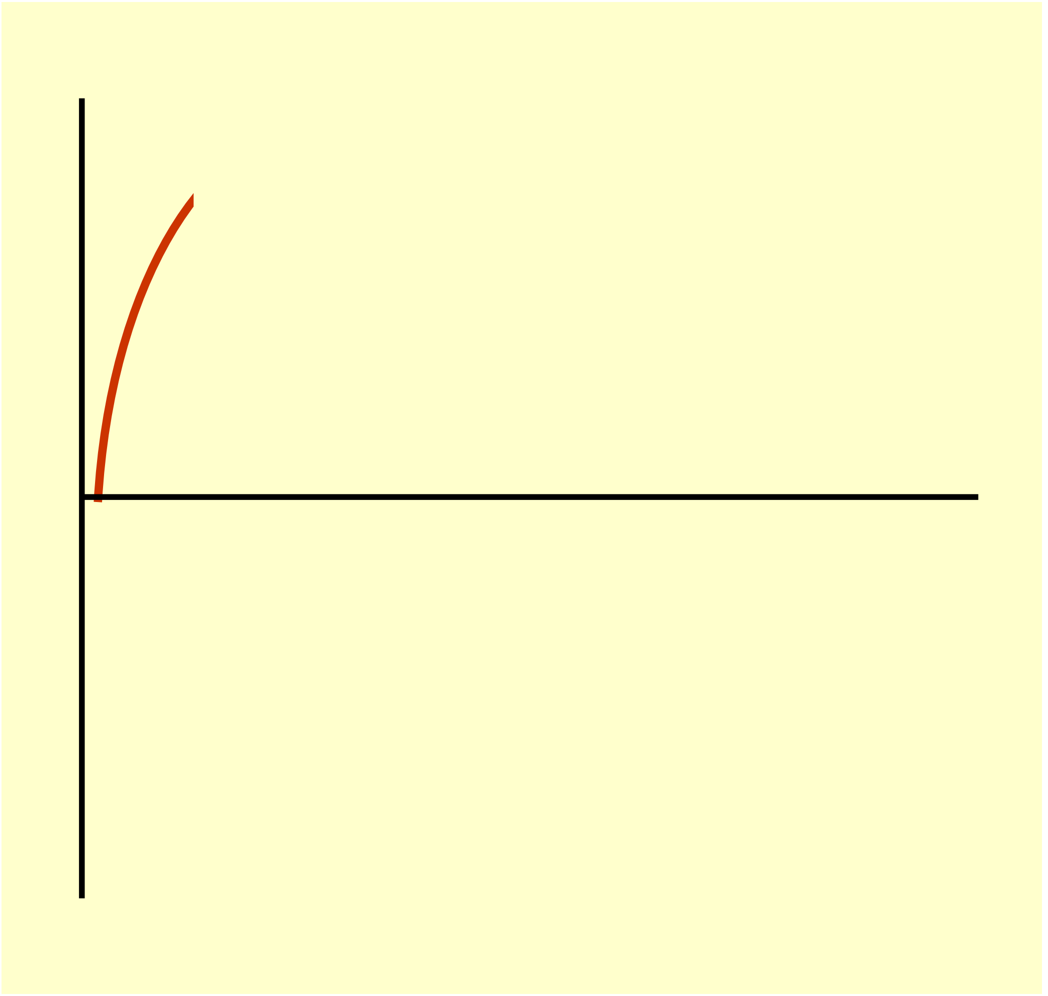


Motion is parallel to the flux.

No voltage is induced.

GENERATING A SINGLE PHASE

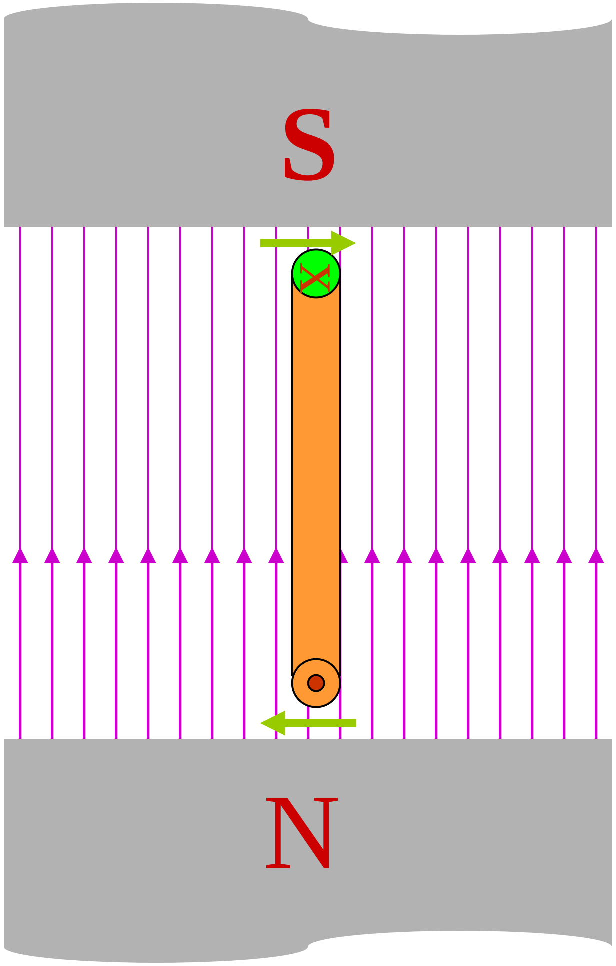


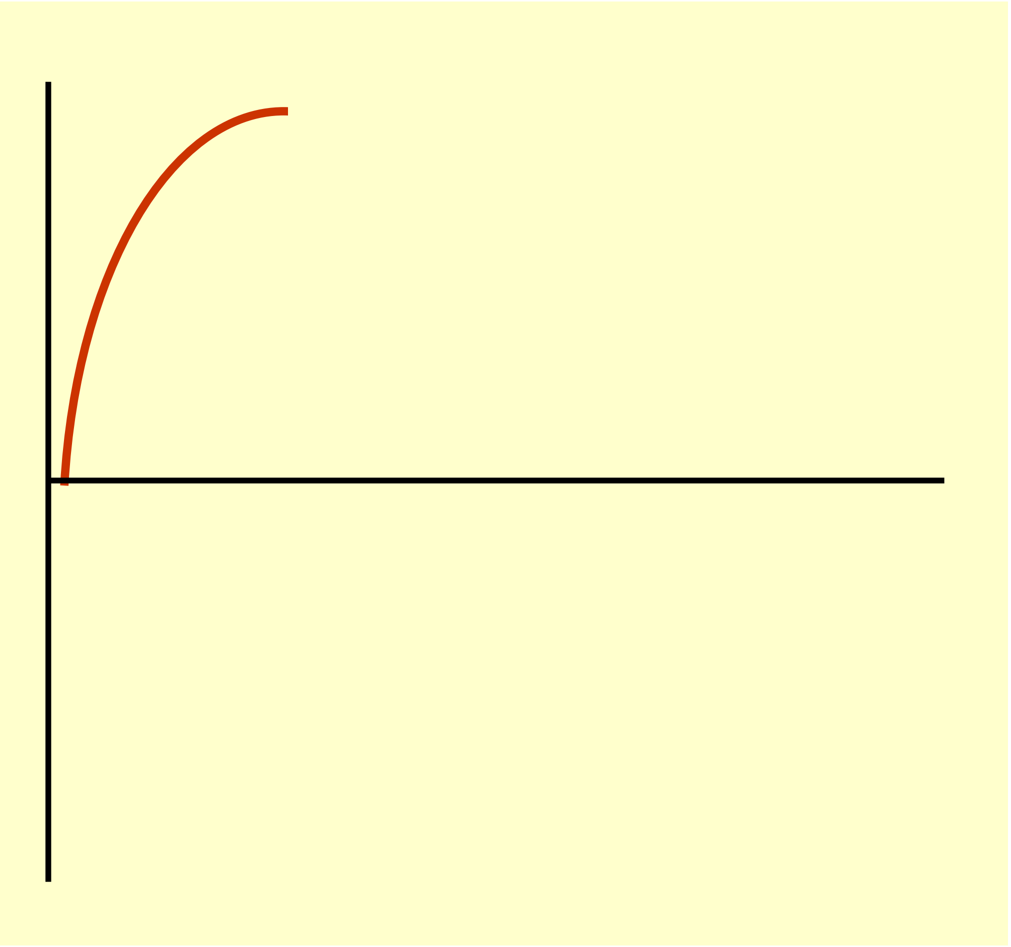


Motion is 45° to flux.

Induced voltage is 0.707 of maximum.

GENERATING A SINGLE PHASE

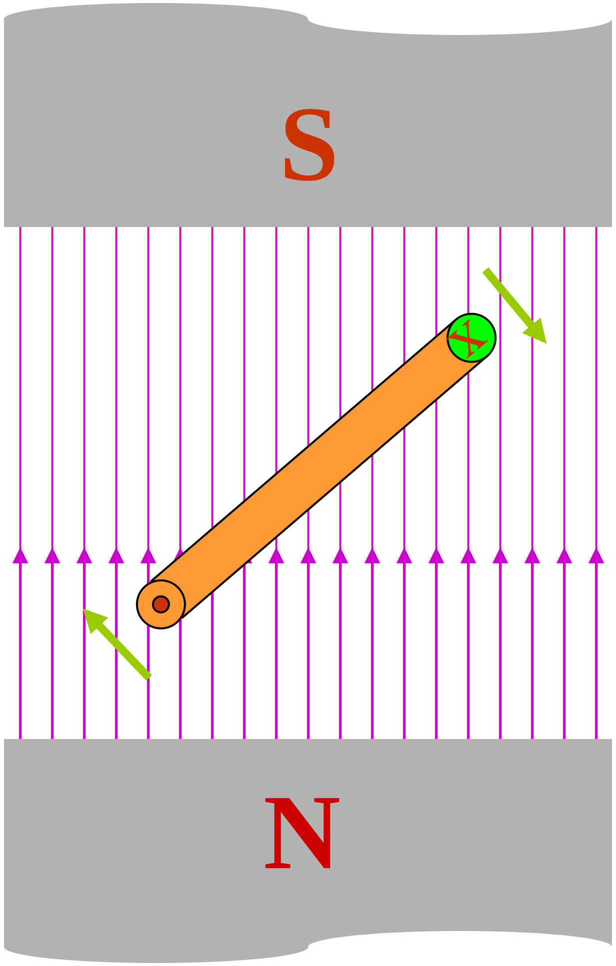


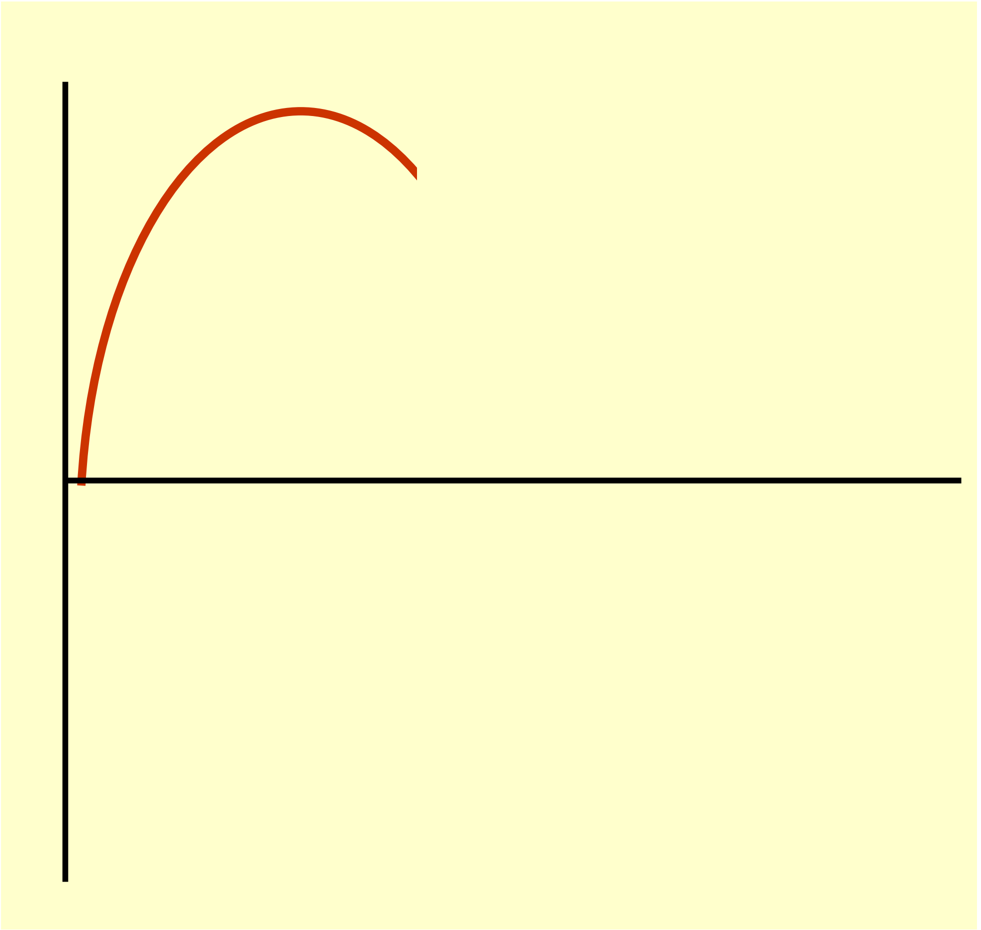


Motion is perpendicular to flux.

Induced voltage is maximum.

GENERATING A SINGLE PHASE





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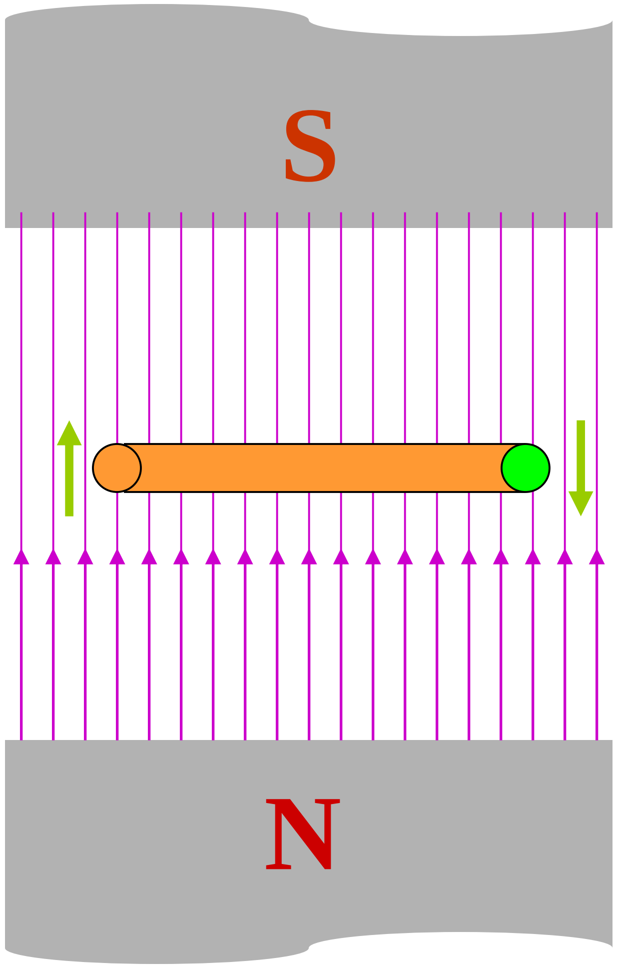
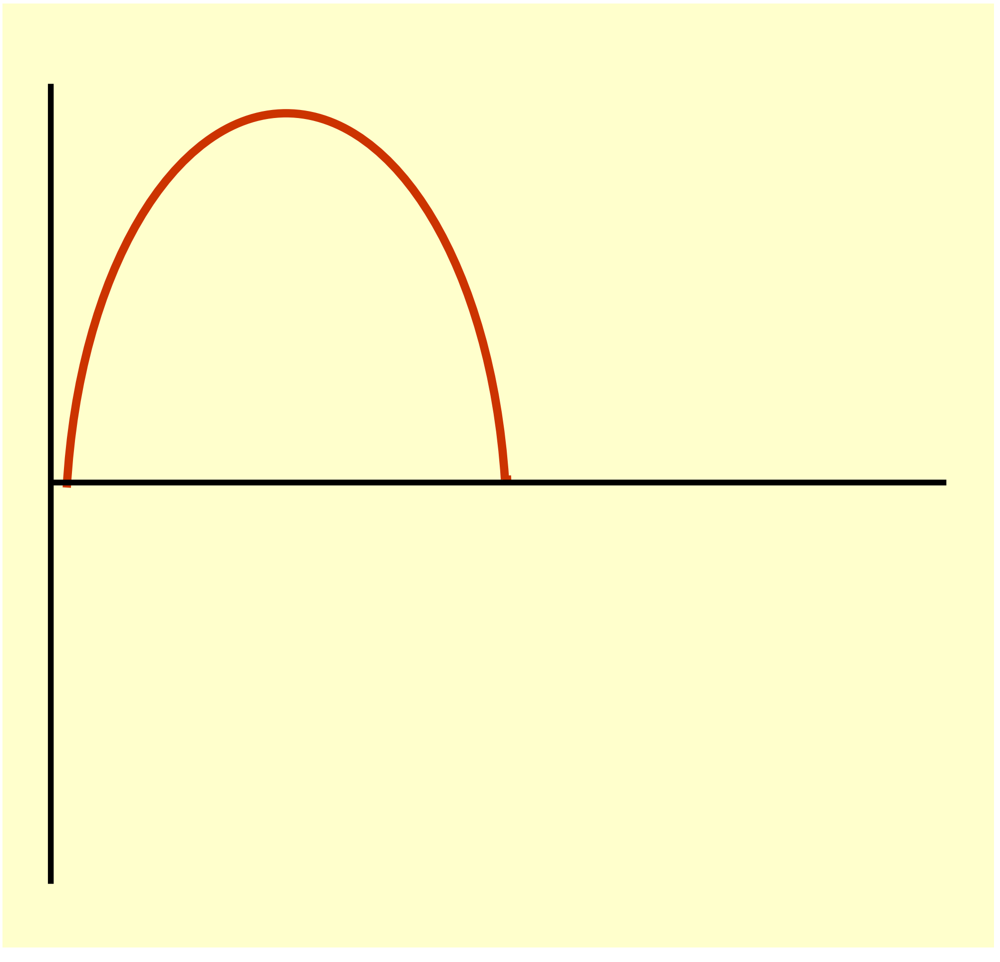
GENERATING A SINGLE PHASE

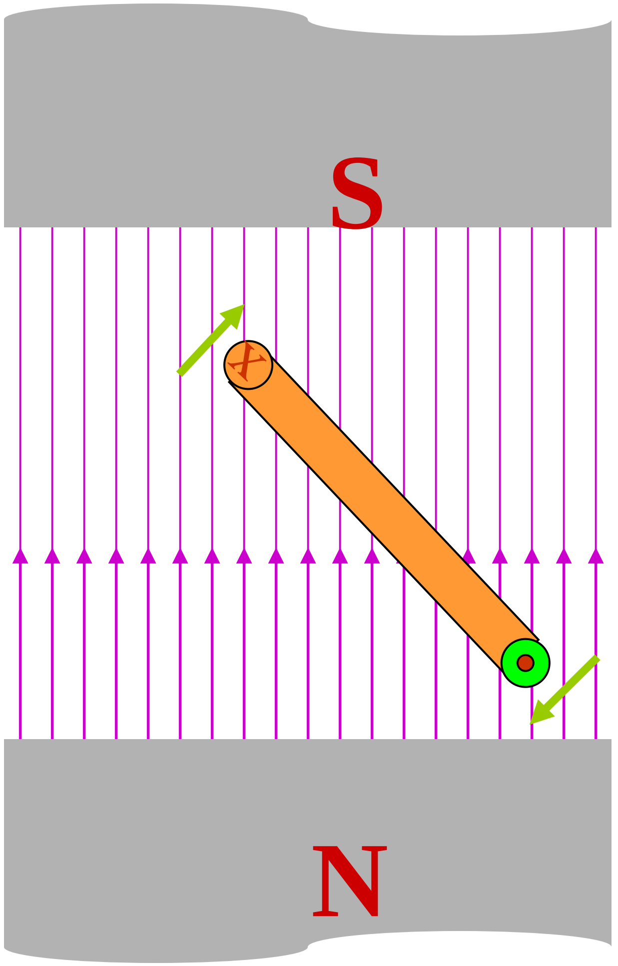


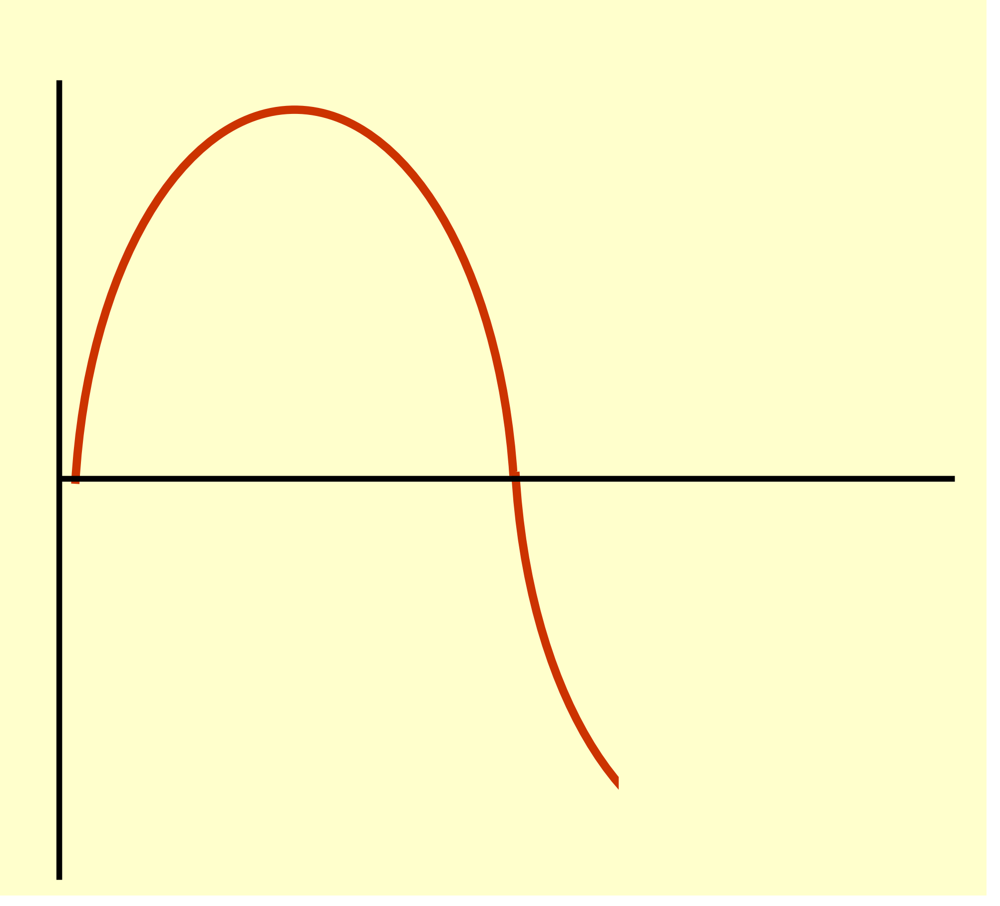


Motion is parallel to flux.

No voltage is induced.

GENERATING A SINGLE PHASE

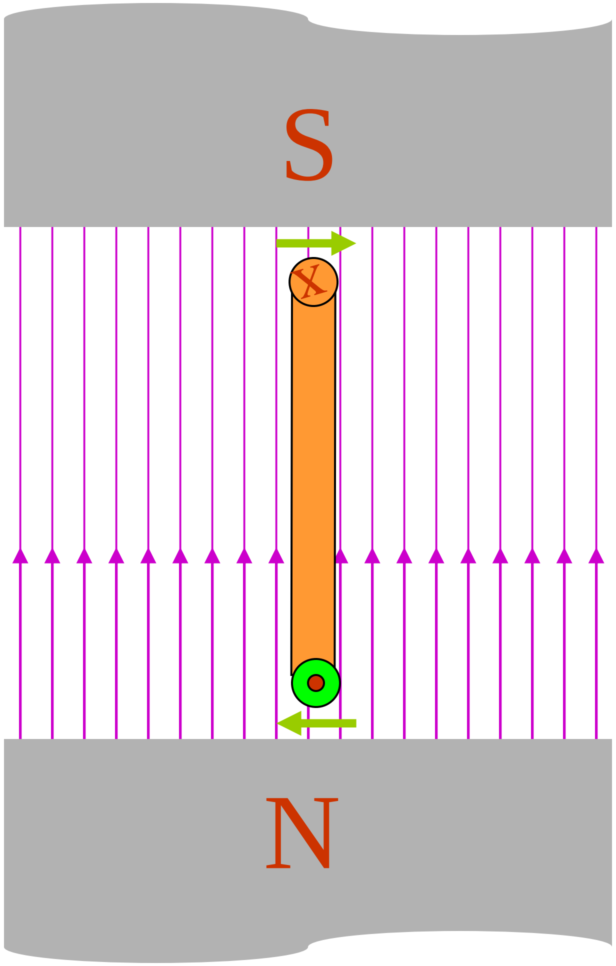


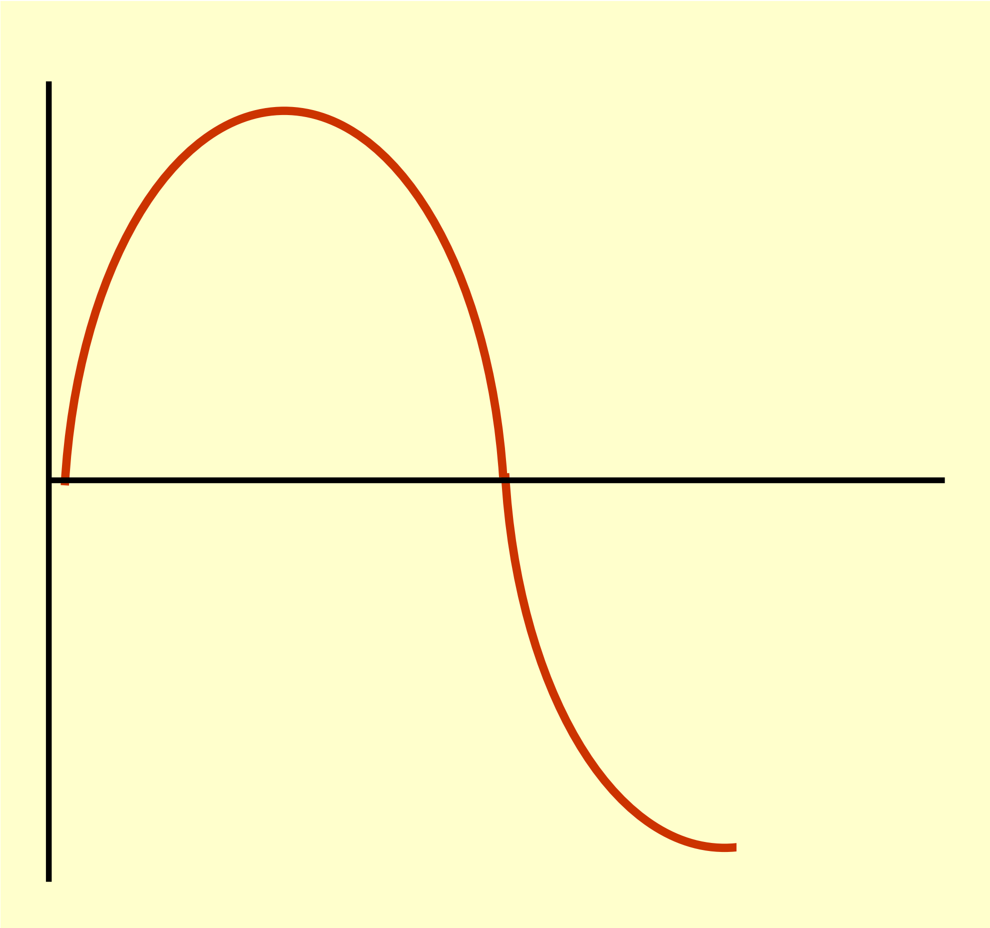


Notice current in the conductor has reversed.

Motion is 45° to flux.
Induced voltage is 0.707 of maximum.

GENERATING A SINGLE PHASE

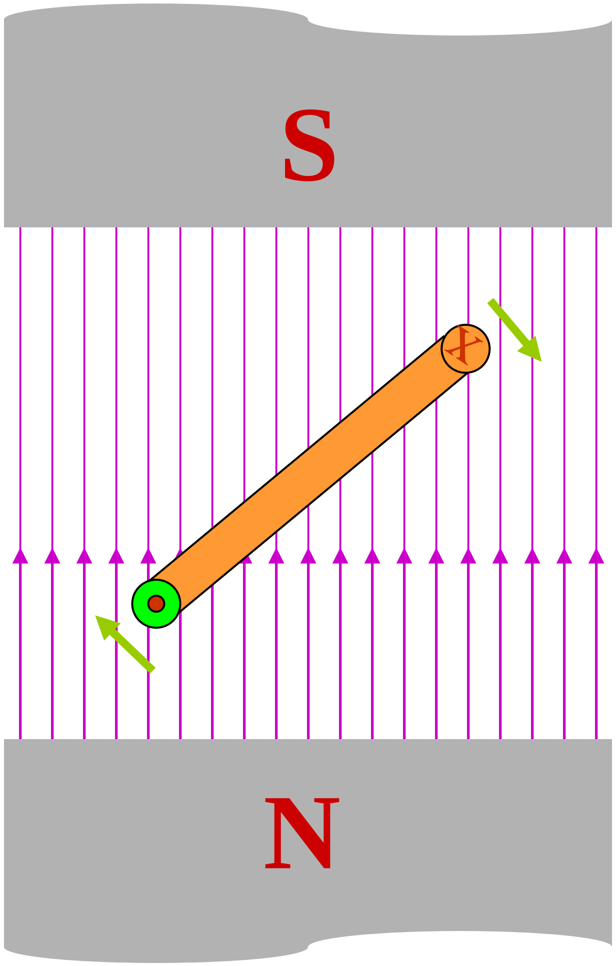


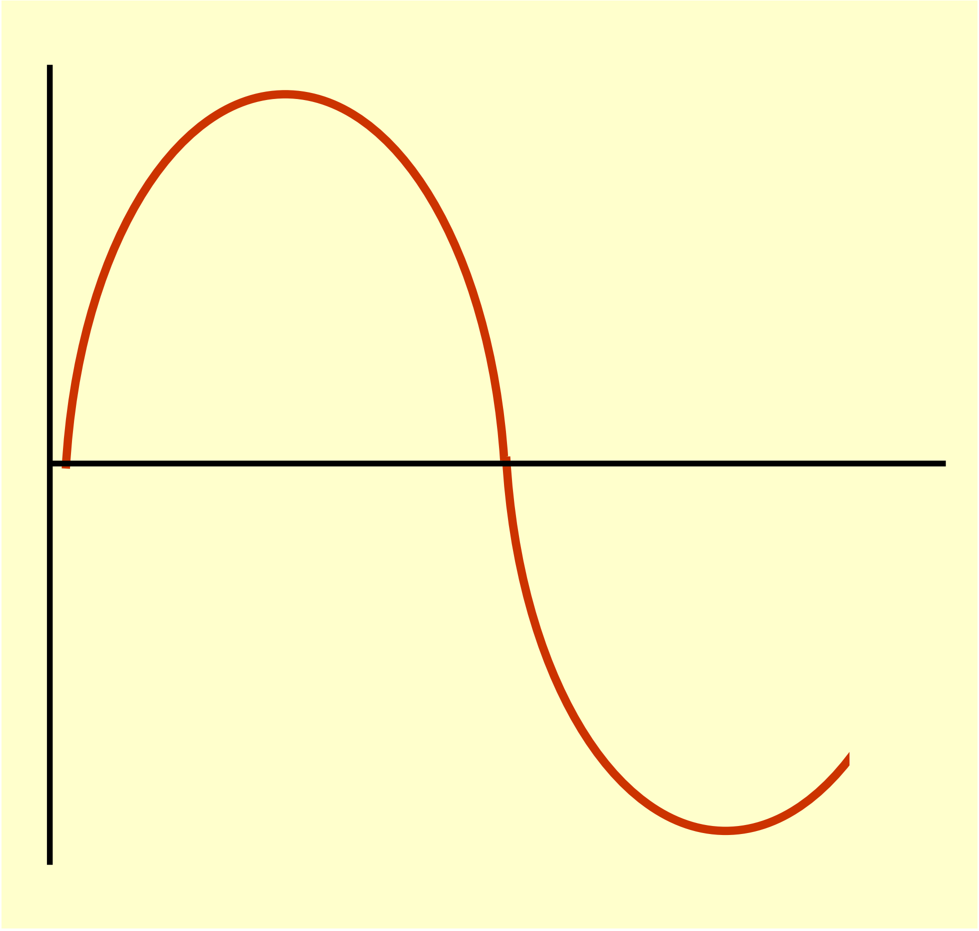


Motion is perpendicular to flux.

Induced voltage is maximum.

GENERATING A SINGLE PHASE





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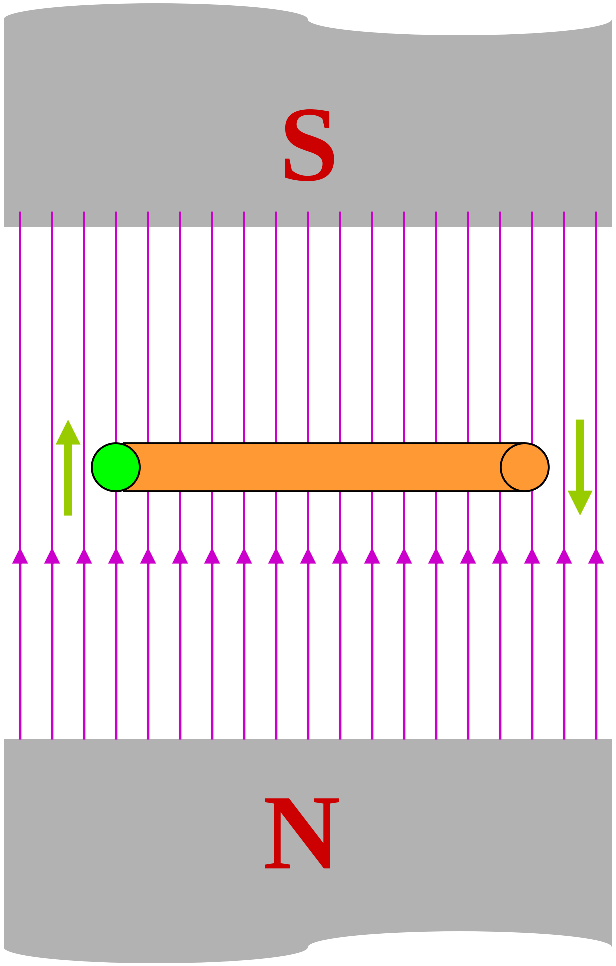
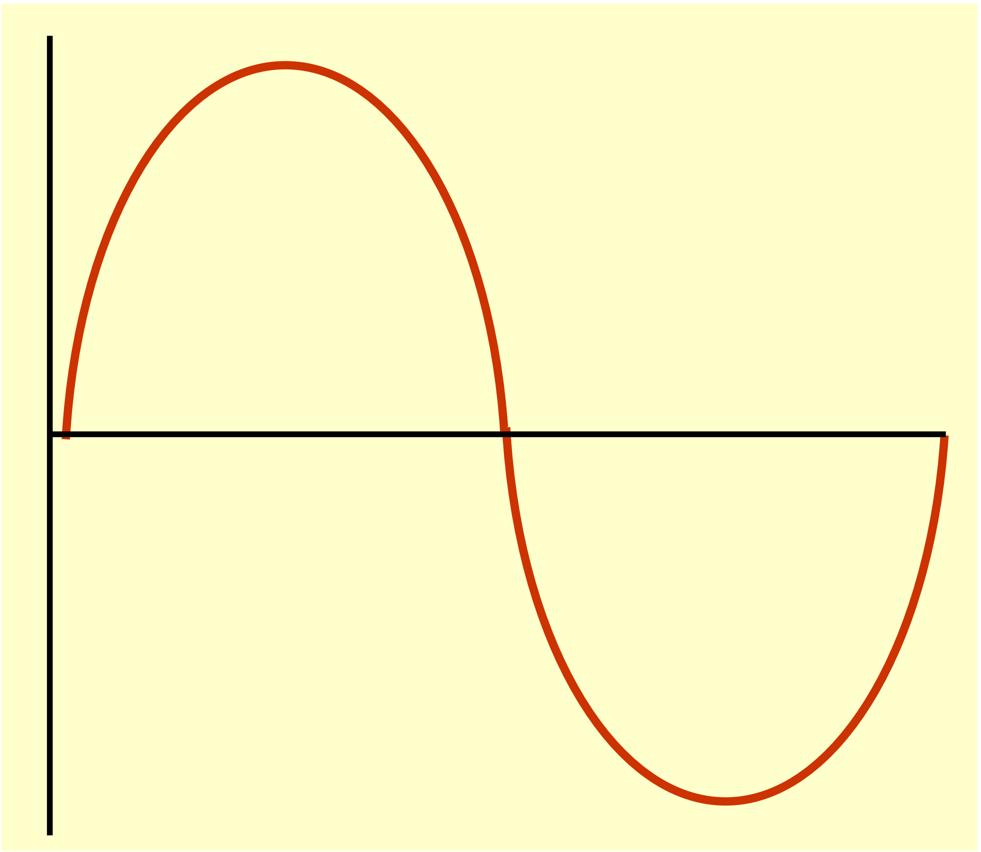
GENERATING A SINGLE PHASE





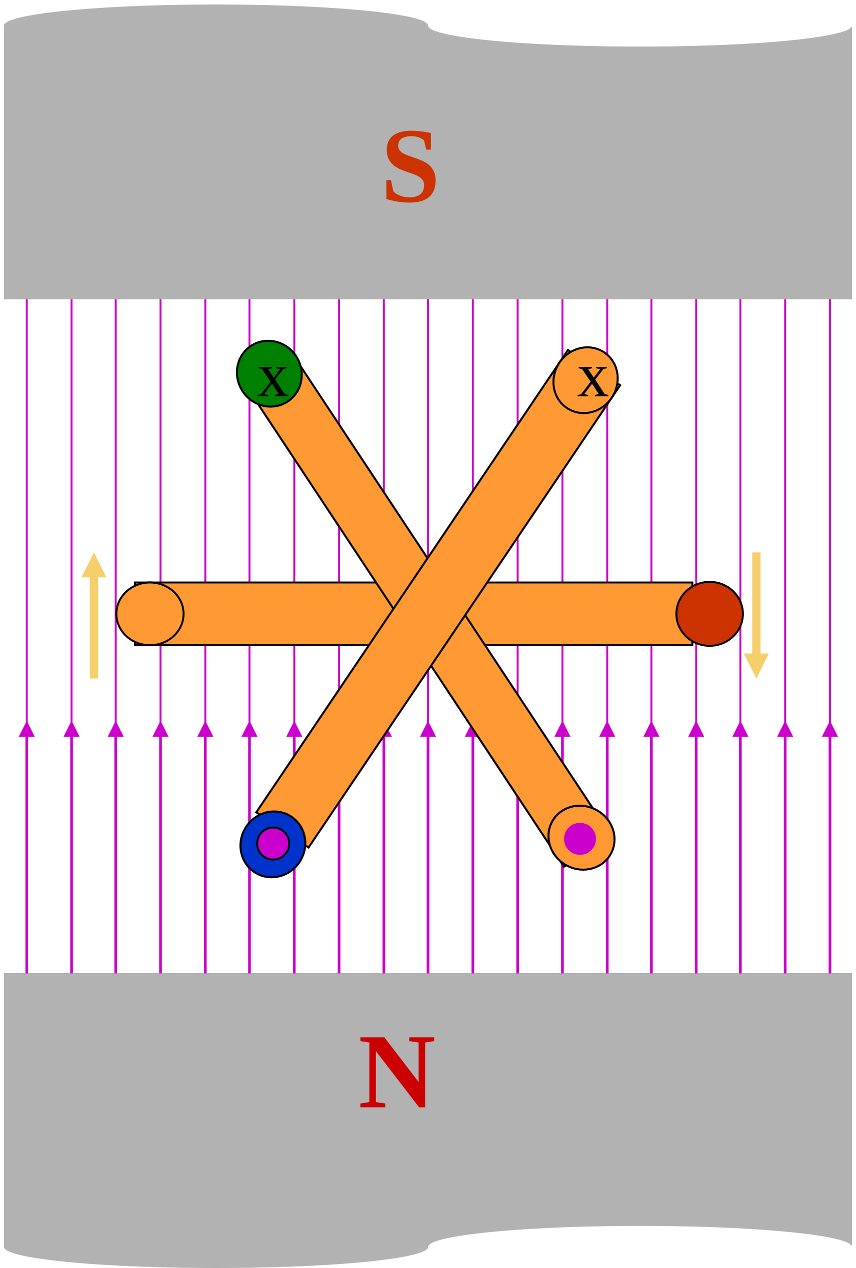
Motion is parallel to flux.

No voltage is induced.

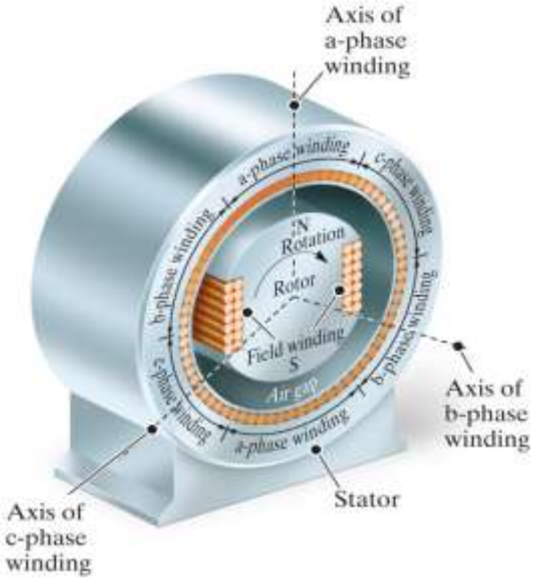
Ready to produce another cycle.

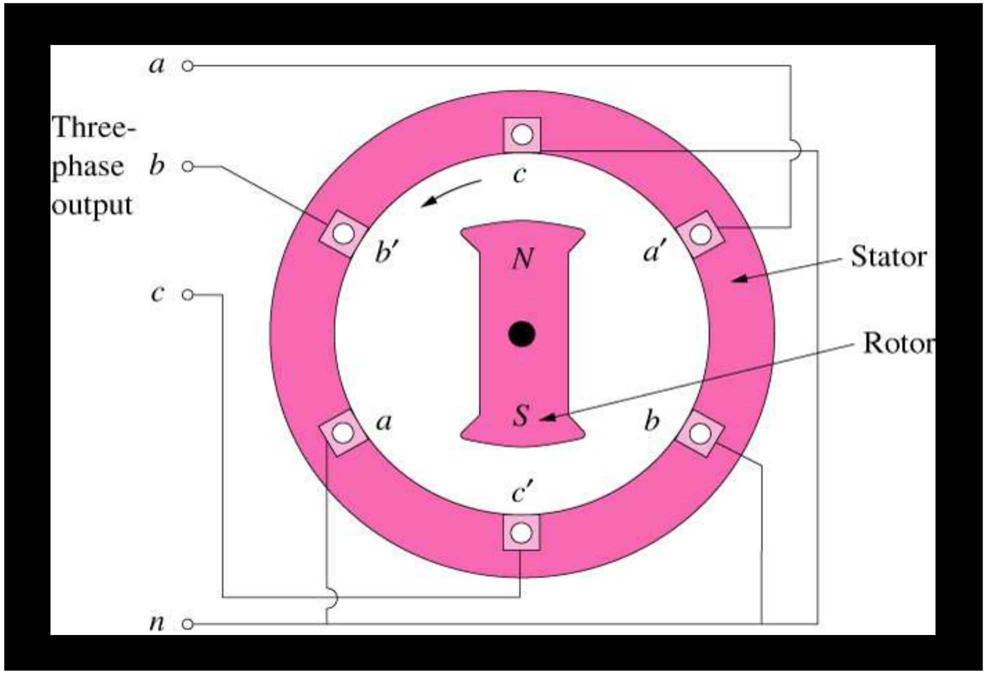
GENERATION OF THREE-PHASE AC

- Three Voltages will be induced across the coils with 120 phase difference



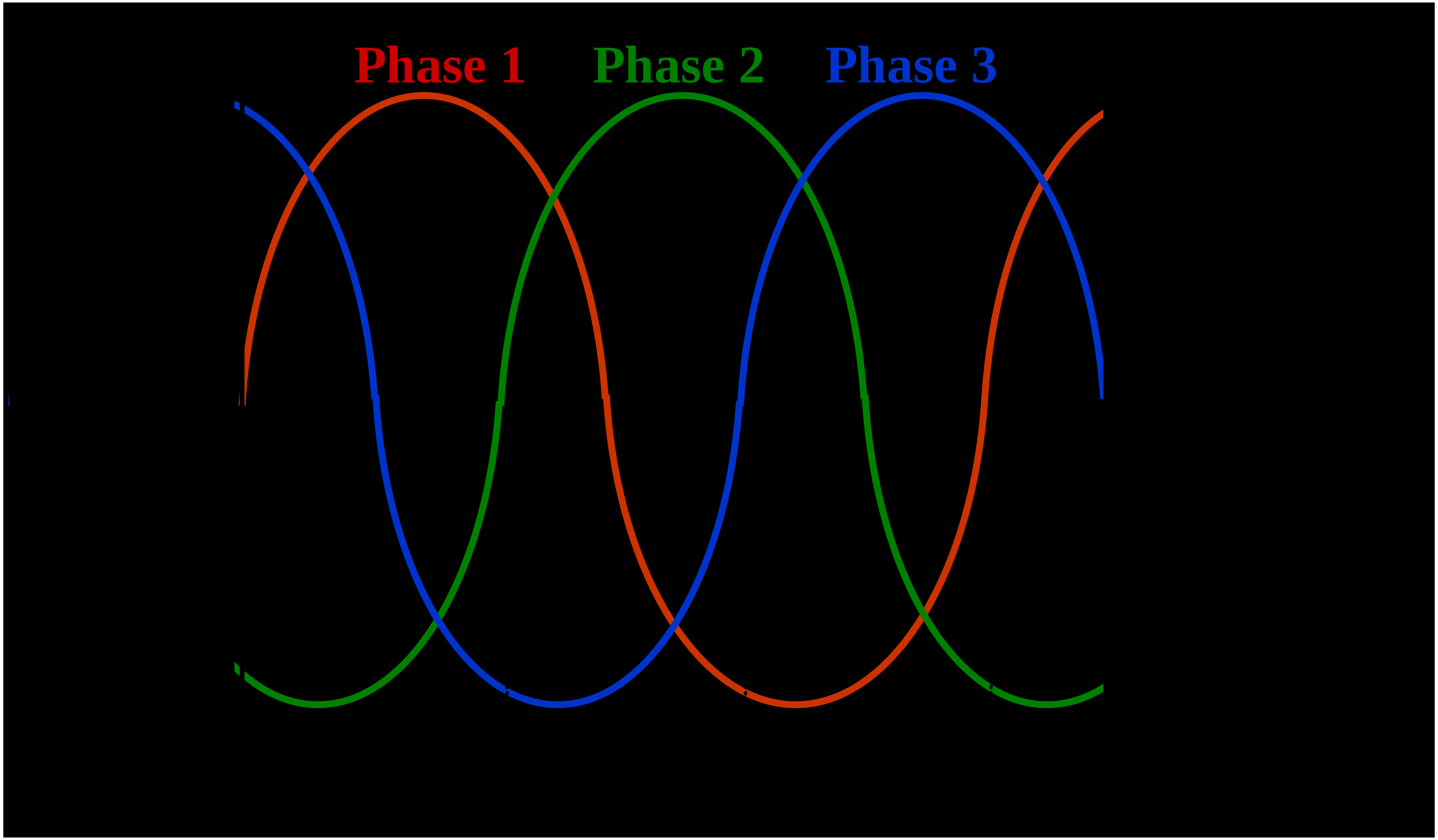
Practical THREE PHASE GENERATOR





- The generator consists of a rotating magnet (rotor) surrounded by a stationary winding (stator).
- Three separate windings or coils with terminals a-a’, b-b’, and c-c’ are physically placed 120° apart around the stator.
- As the rotor rotates, its magnetic field cuts the flux from the three coils and induces voltages in the coils.
- The induced voltage have equal magnitude but out of phase by 120°.

THREE-PHASE WAVEFORM



Phase 2 lags phase 1 by 120° . Phase 2 leads phase 3 by 120° .

Phase 3 lags phase 1 by 240° . Phase 1 leads phase 3 by 240° .

WHY WE STUDY 3 PHASE SYSTEM ?

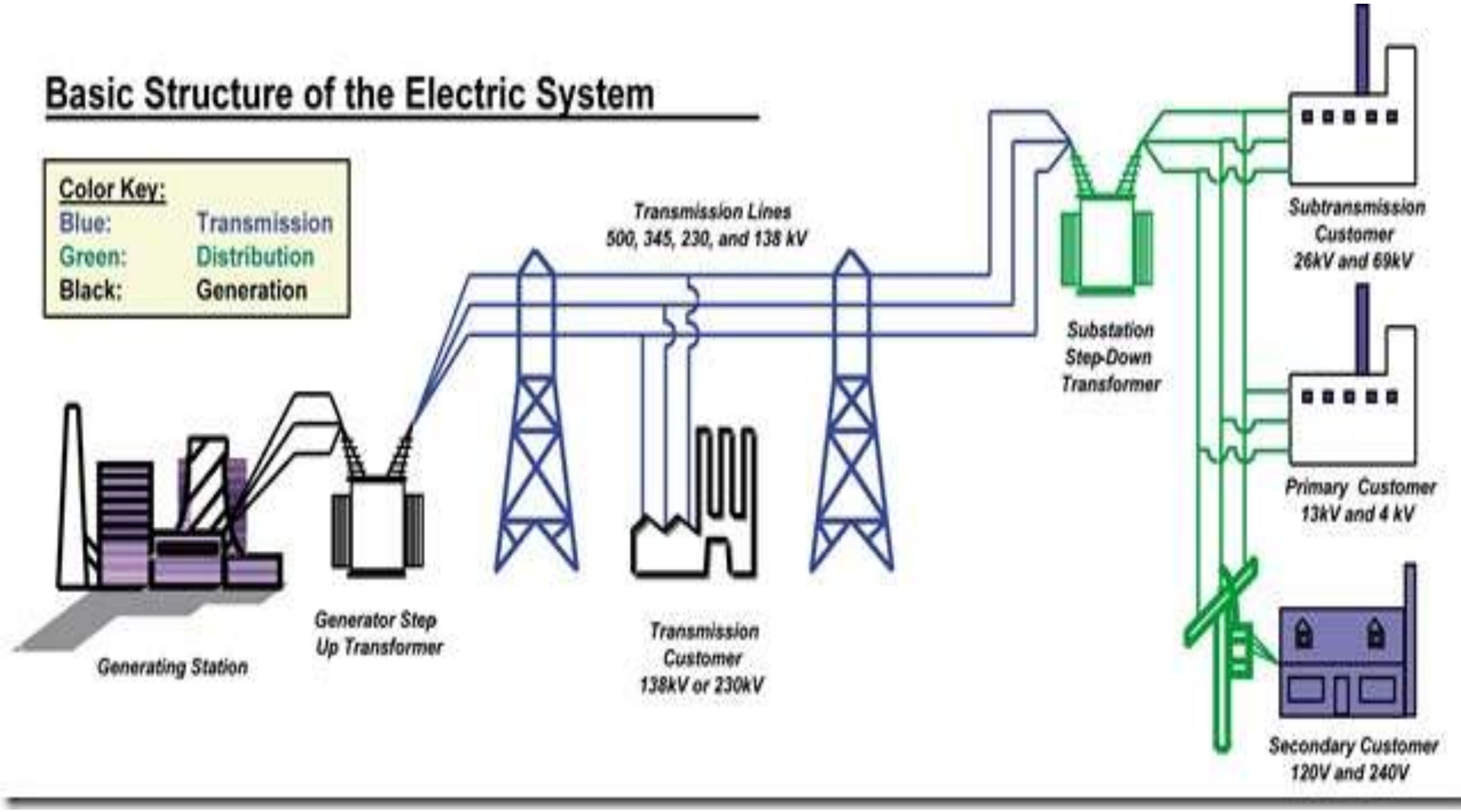
- ALL electric power system in the world used 3-phase system to GENERATE, TRANSMIT and DISTRIBUTE
  - ✓ One phase, two phase, or three phase ican be taken from three phase system rather than generated independently.

- Instantaneous power is constant (not pulsating).– thus smoother rotation of electrical machines
  - ✓ High power motors prefer a steady torque

- More economical than single phase – less wire for the same power transfer
  - ✓ The amount of wire required for a three phase system is less than required for an equivalent single phase system.

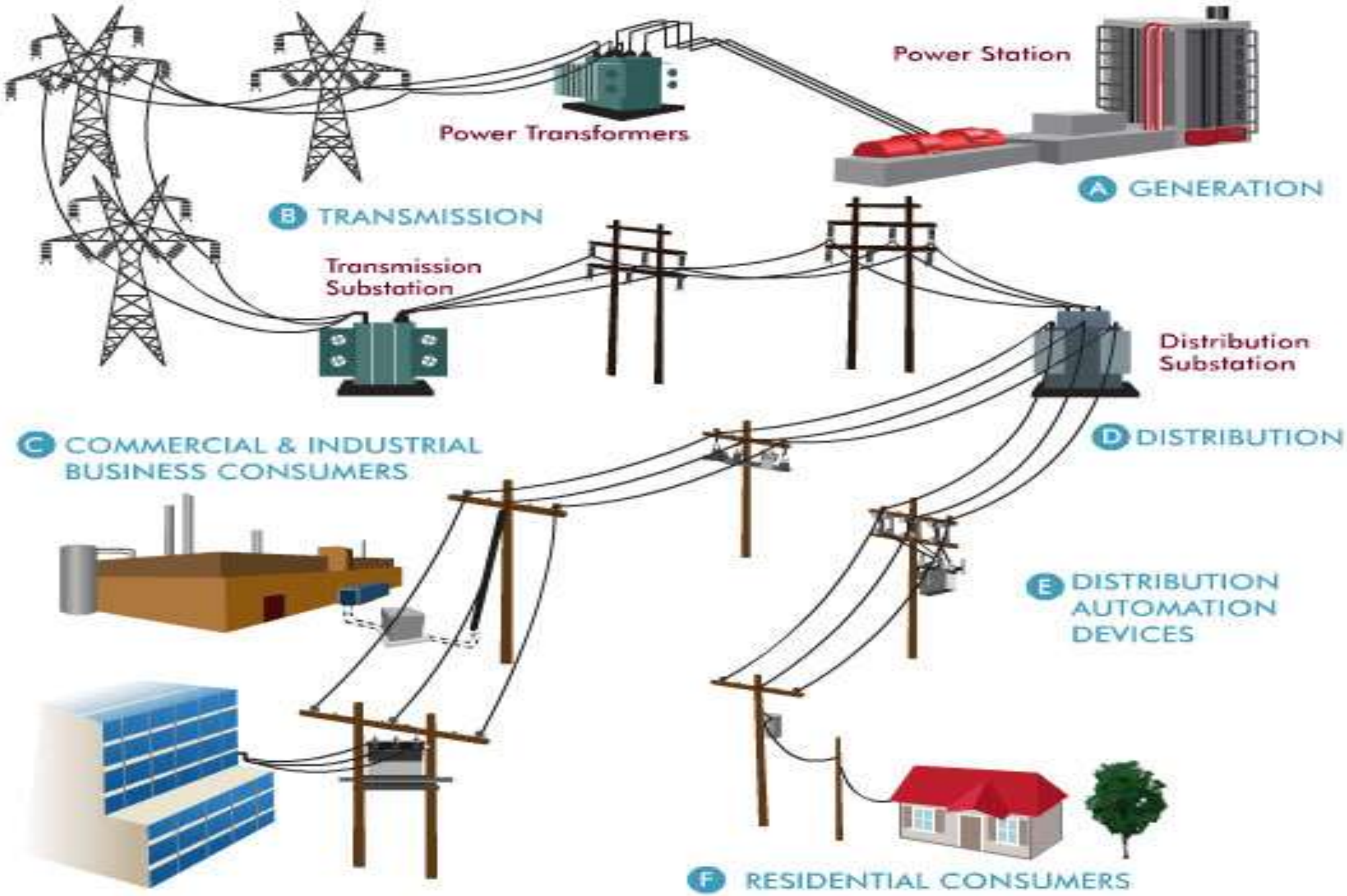
3-phase systems

Generation, Transmission and Distribution



3-phase systems

Generation, Transmission and Distribution

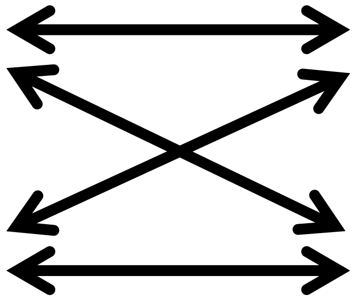


Y and Δ connections

Balanced 3-phase systems can be considered as 3 equal single phase voltage sources connected either as Y or Delta (Δ) to 3 single three loads connected as either Y or Δ

SOURCE CONNECTIONS **LOAD CONNECTIONS**

Y connected source **Y connected load**

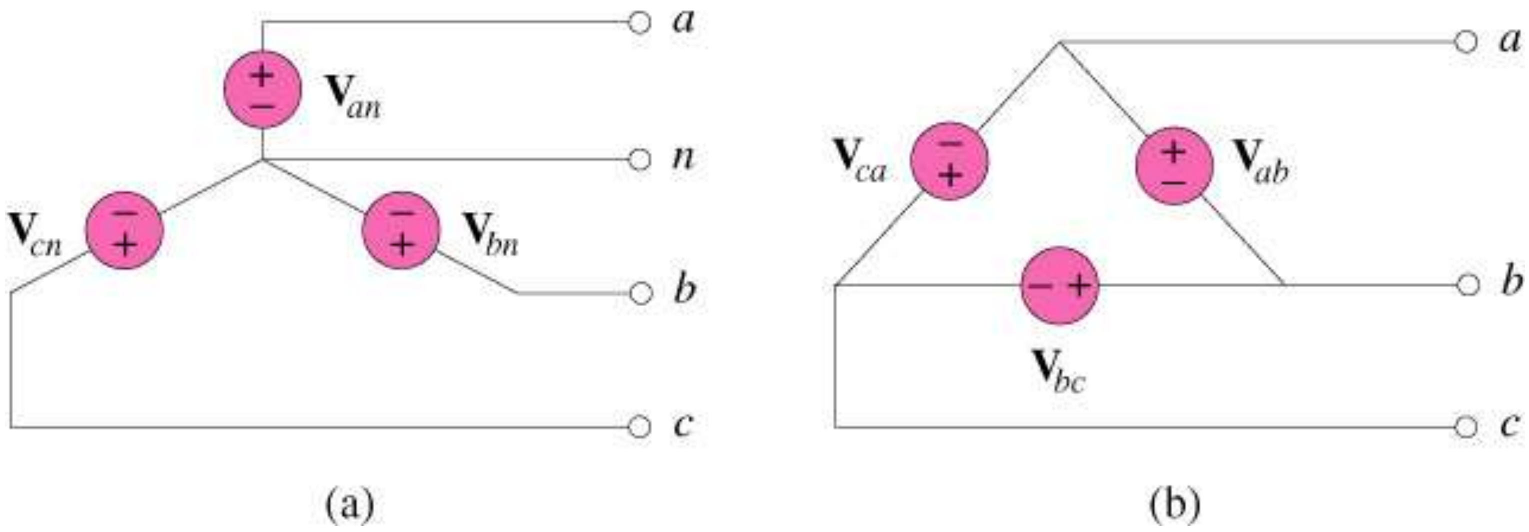


Δ connected source **Δ connected load**

Y-Y **Y-Δ** **Δ-Y** **Δ-Δ**

Balance Three-Phase Sources

Two possible configurations:

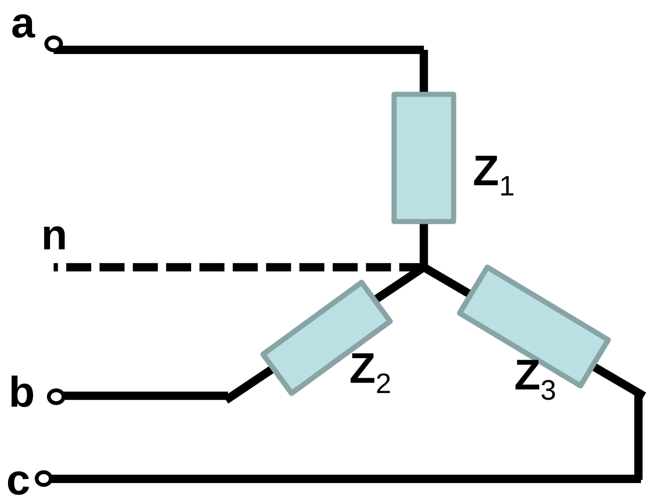


Three-phase voltage sources: (a) Y-connected ; (b) Δ-connected

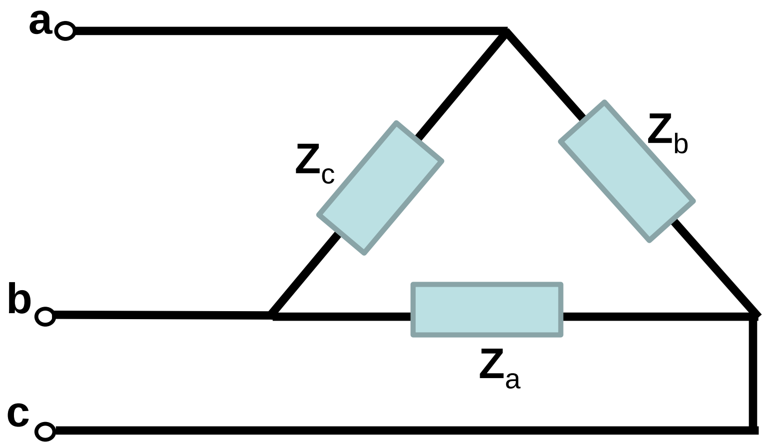
Balanced 3-phase systems

LOAD CONNECTIONS

Y connection



Δ connection



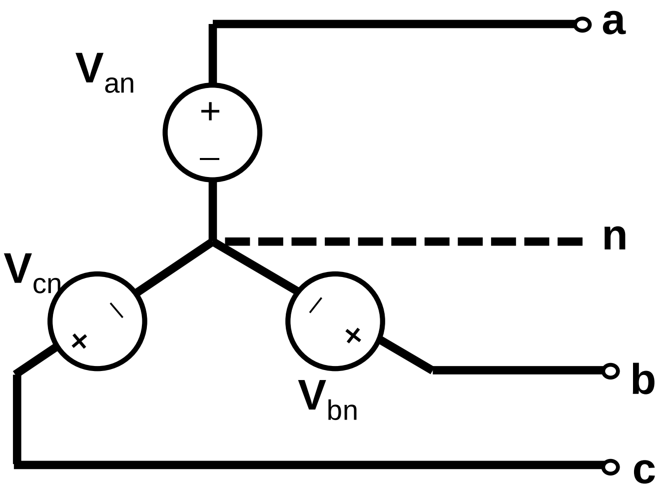
Balanced load:

$$Z_1 = Z_2 = Z_3 = Z_Y \qquad Z_a = Z_b = Z_c = Z_\Delta \qquad Z_Y = \frac{Z_\Delta}{3}$$

Unbalanced load: each phase load may not be the same.

Phase Sequence

The ***phase sequence*** is the time order in which the voltages pass through their respective maximum values.

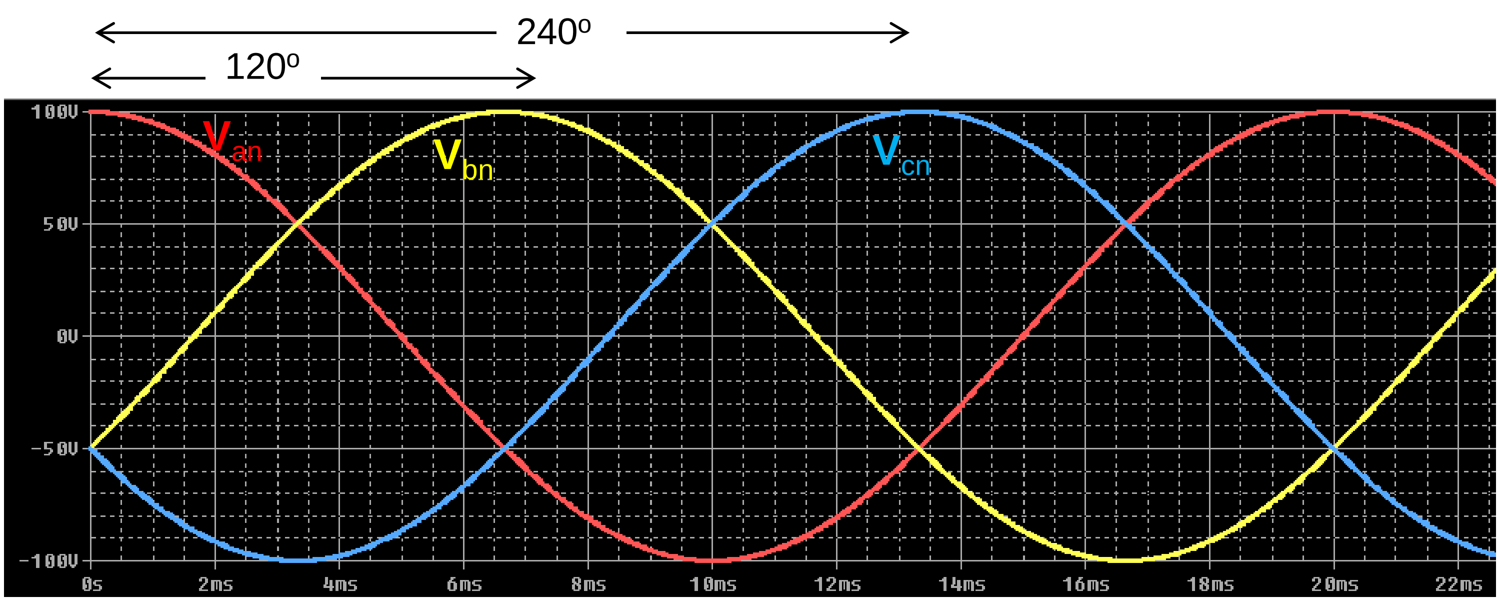


$$\Rightarrow \mathbf{V}_{an} = V_p \angle 0^o \qquad v_{an}(t) = \sqrt{2} V_p \cos(\omega t)$$

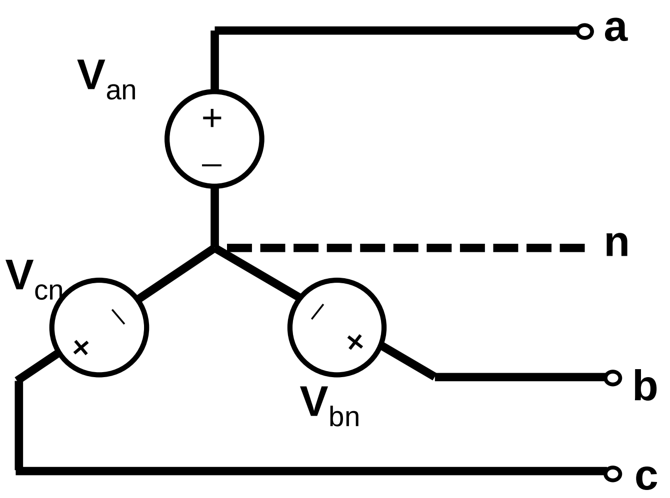
$$\Rightarrow \mathbf{V}_{bn} = V_p \angle -120^o \qquad v_{bn}(t) = \sqrt{2} V_p \cos(\omega t - 120^o)$$

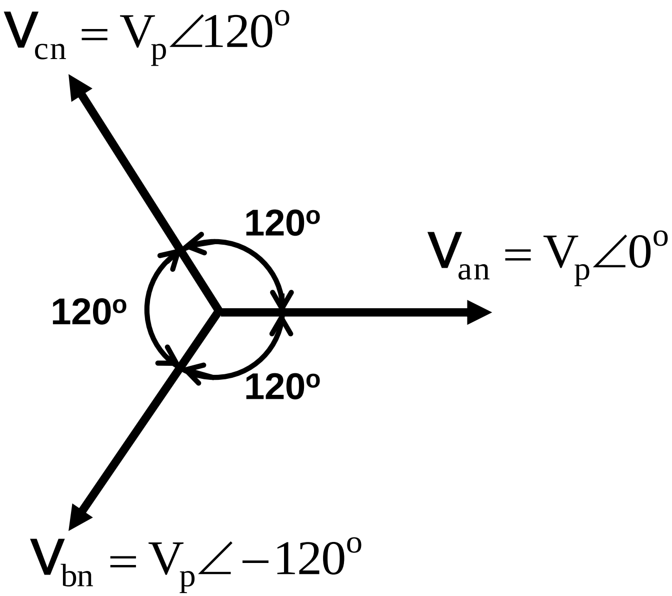
$$\Rightarrow \mathbf{V}_{cn} = V_p \angle 120^o \qquad v_{cn}(t) = \sqrt{2} V_p \cos(\omega t + 120^o)$$

RMS phasors !



Phase Sequence





Phase sequence : $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$ by 120^o and $\mathbf{V}_{bn}$ leads $\mathbf{V}_{cn}$ by 120^o

→This is a known as **abc sequence** or **positive sequence**

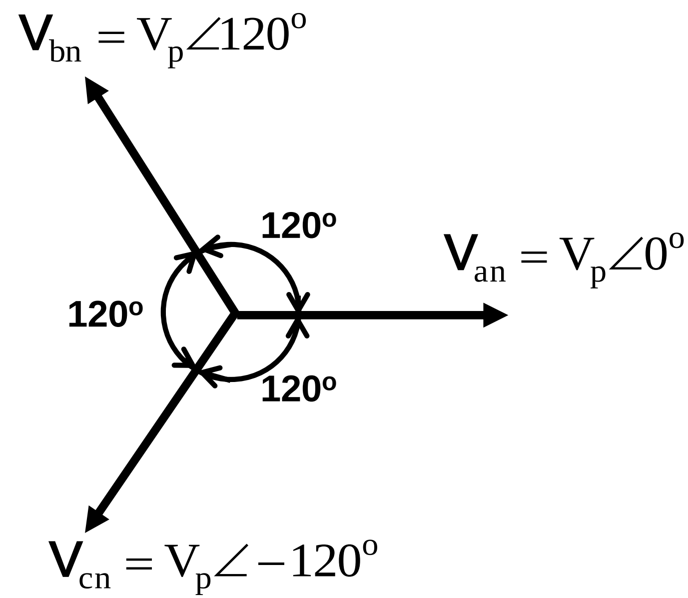
Phase Sequence

What if different phase sequence?

$$v_{an}(t) = \sqrt{2}V_p\cos(\omega t) \Rightarrow \mathbf{V}_{an} = V_p\angle 0^o$$

$$v_{cn}(t) = \sqrt{2}V_p\cos(\omega t - 120^o) \Rightarrow \mathbf{V}_{cn} = V_p\angle -120^o$$

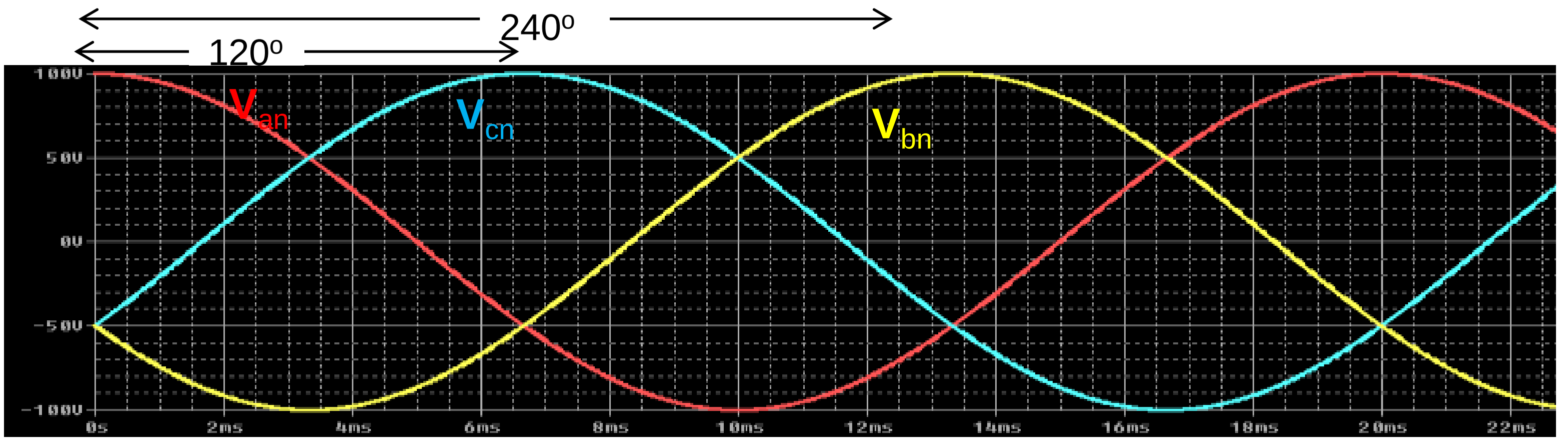
$$v_{bn}(t) = \sqrt{2}V_p\cos(\omega t + 120^o) \Rightarrow \mathbf{V}_{bn} = V_p\angle 120^o$$



RMS phasors !

Phase sequence : $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120º and $\mathbf{V}_{cn}$ leads $\mathbf{V}_{bn}$ by 120º

→This is a known as **acb sequence** or **negative sequence**



Example

Determine the phase sequence of the set of voltages.

$$v_{an} = 200\cos(\omega t + 10°)$$

$$v_{bn} = 200\cos(\omega t - 230°)$$

$$v_{cn} = 200\cos(\omega t - 110°)$$

Solution:

The voltages can be expressed in phasor form as

$$\mathrm{V}_{an} = (200/\sqrt{2})\angle 10° \text{ V}$$

$$\mathrm{V}_{bn} = (200/\sqrt{2})\angle -230° \text{ V}$$

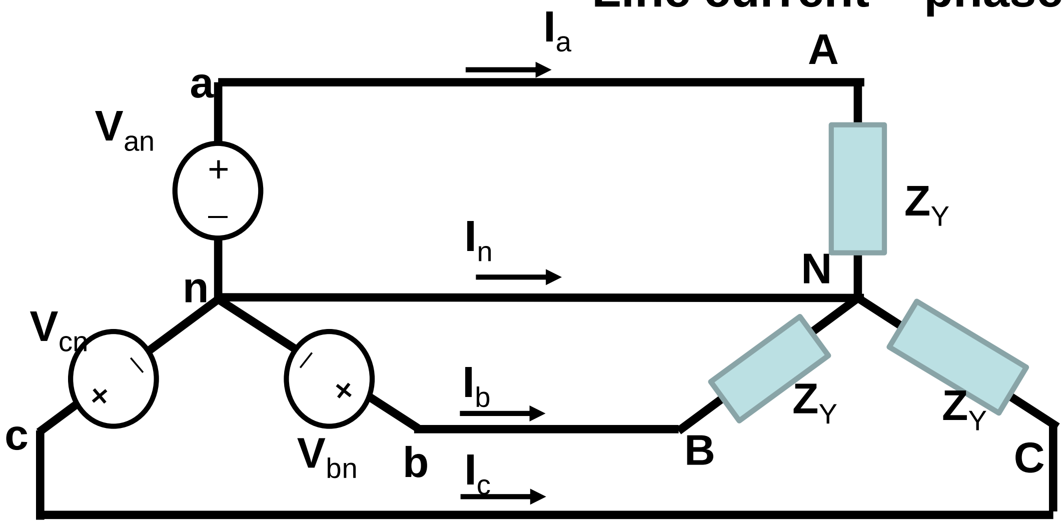
$$\mathrm{V}_{cn} = (200/\sqrt{2})\angle -110° \text{ V}$$

We notice that $\mathbf{V_{an}}$ leads $\mathbf{V_{cn}}$ by 120° and $\mathbf{V_{cn}}$ in turn leads $\mathbf{V_{bn}}$ by 120°.

Hence, we have an **acb** sequence.

Balanced 3-phase Y-Y

Line current = phase current



$$V_{an} = V_p\angle 0^o$$

$$V_{bn} = V_p\angle -120^o$$

$$V_{cn} = V_p\angle 120^o$$

Phase voltages

measured between the <u>neutral</u> and any line (line to neutral voltage)

$$I_a = \frac{V_p\angle 0^o}{Z_Y}$$

$$I_b = \frac{V_p\angle -120^o}{Z_Y}$$

$$I_c = \frac{V_p\angle 120^o}{Z_Y}$$

line currents

$$\therefore I_a + I_b + I_c = I_n = 0$$

The wire connecting n and N can be removed !

$$V_{ab} = V_a - V_b = V_a - V_b + V_n - V_n =$$
$$= V_{an} + V_{nb} = V_p\angle 0^o + V_p\angle 60^o$$
$$= \sqrt{3}V_p\angle 30^o$$

$$V_{bc} = V_{bn} + V_{nc}$$
$$= \sqrt{3}V_p\angle -90^o$$

$$V_{ca} = V_{cn} + V_{na}$$
$$= \sqrt{3}V_p\angle 150^o$$

line-line voltages OR Line voltages

Balanced 3-phase systems

Balanced Y-Y Connection

$$
\begin{aligned}
\mathbf{V}_{ab} &= V_{an} + V_{nb} \\
&= V_p\angle 0^o + V_p\angle 60^o \\
&= \sqrt{3}V_p\angle 30^o
\end{aligned}
$$

Balanced 3-phase systems

Balanced Y-Y Connection

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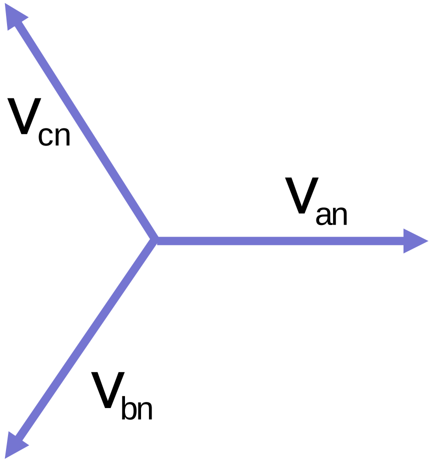

Balanced 3-phase systems

Balanced Y-Y Connection

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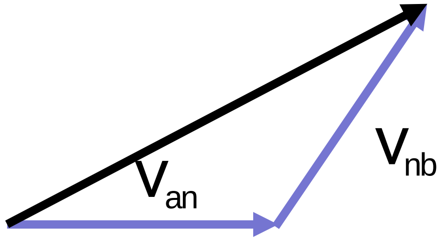

Balanced 3-phase systems

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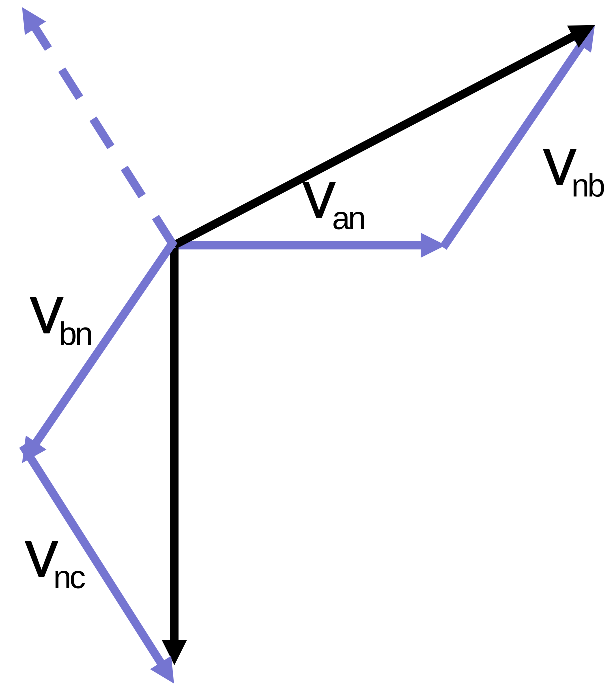

Balanced 3-phase systems

Balanced Y-Y Connection

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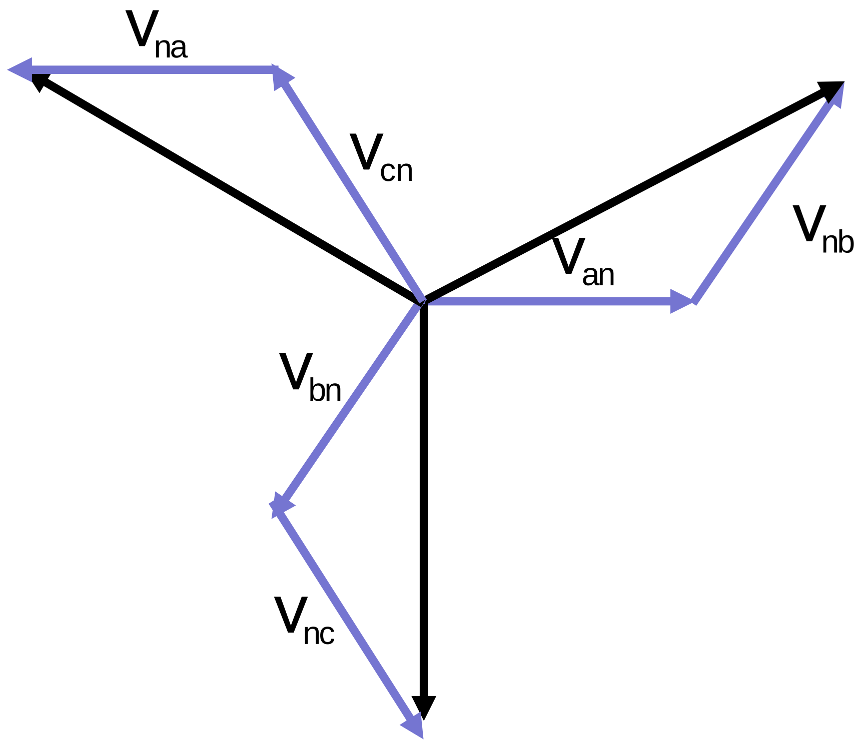
$$V_{ca} = V_{cn} + V_{na}$$
$$= \sqrt{3} V_p \angle 150^o$$



Balanced 3-phase systems

Balanced Y-Y Connection

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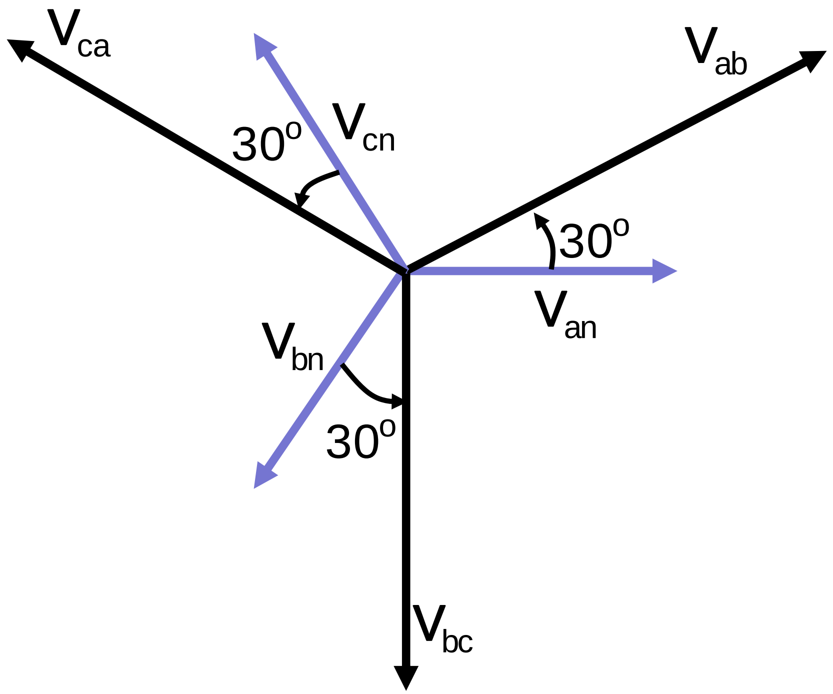
$$V_{ca} = V_{cn} + V_{na}$$
$$= \sqrt{3}V_p\angle 150^o$$



$$V_L = \sqrt{3}V_p$$

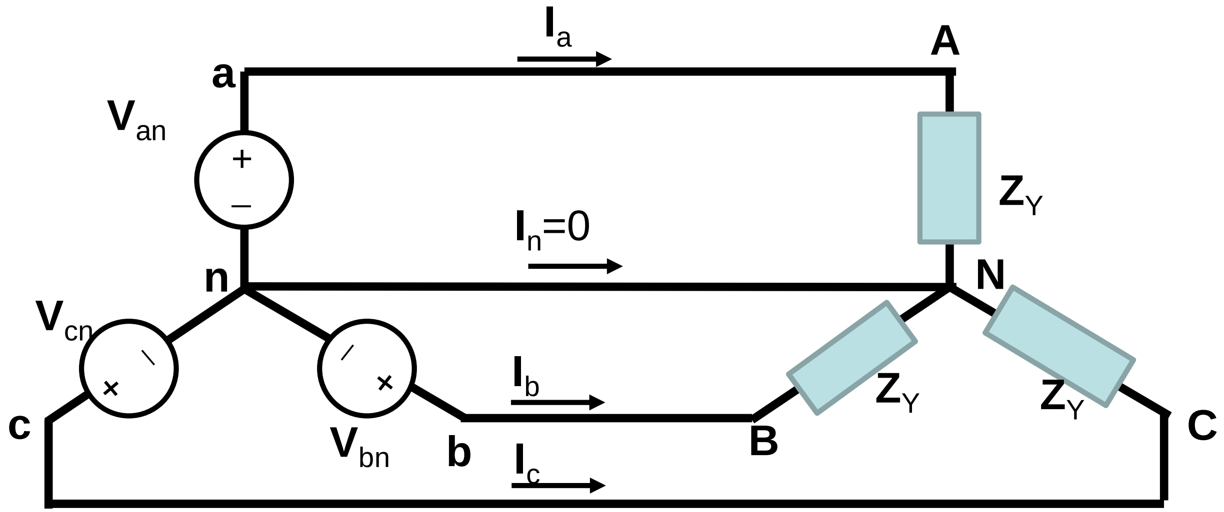
where $V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$ and $V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$

Line voltage LEADS phase voltage by 30^o

Balanced 3-phase systems

Balanced Y-Y Connection

For a **balanced Y-Y** connection, analysis can be performed using an equivalent per-phase circuit: e.g. for phase A:



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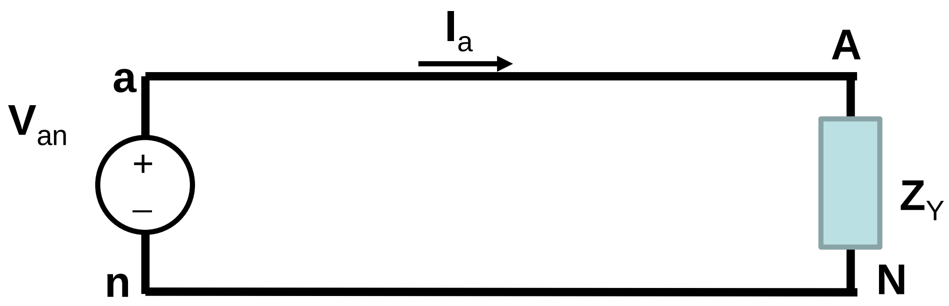


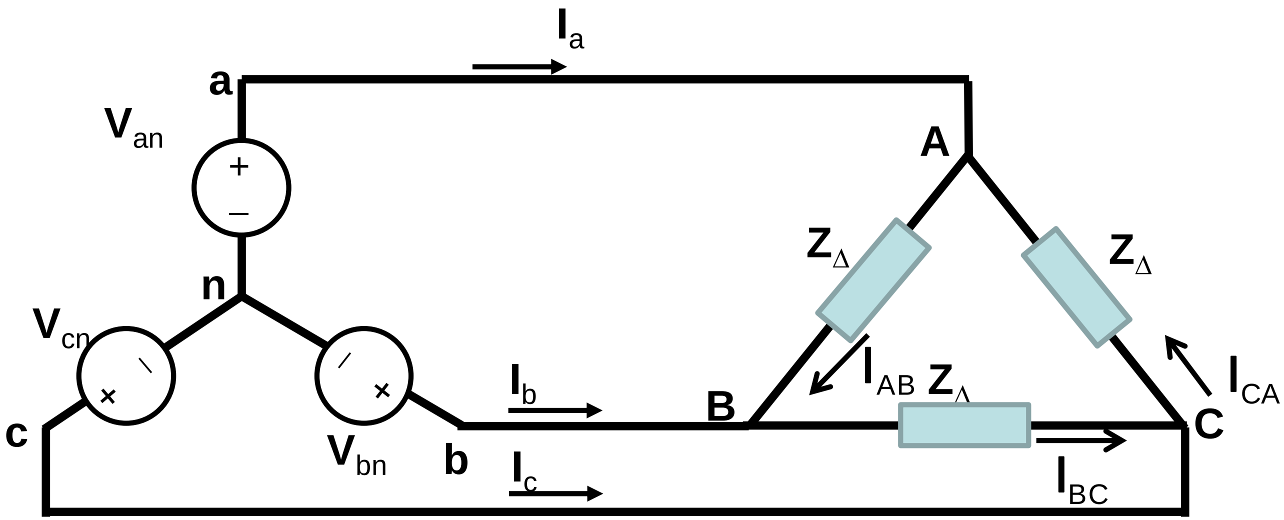
$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

Based on the sequence, the other line currents can be obtained from:

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^o \qquad \mathbf{I}_c = \mathbf{I}_a \angle 120^o$$

Balanced 3-phase systems

Balanced Y-Δ Connection



$$V_{an} = V_p\angle 0^o$$

$$V_{bn} = V_p\angle -120^o$$

$$V_{cn} = V_p\angle 120^o$$

$$V_{ab} = \sqrt{3}V_p\angle 30^o = V_{AB}$$

$$V_{bc} = \sqrt{3}V_p\angle -90^o = V_{BC}$$

$$V_{ca} = \sqrt{3}V_p\angle 150^o = V_{CA}$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Phase currents

Using KCL,

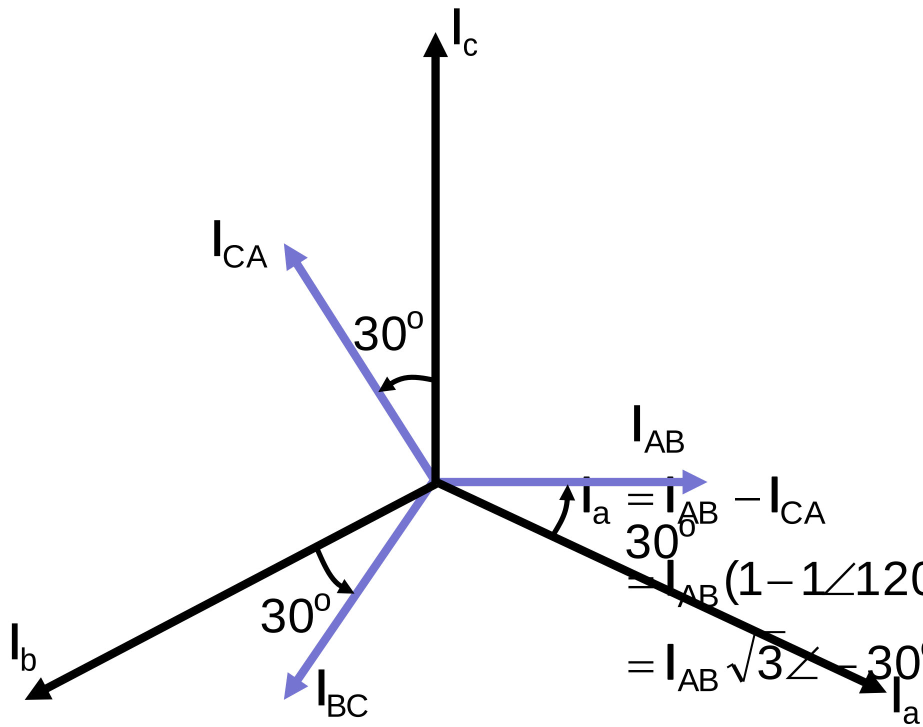
$$I_a = I_{AB} - I_{CA} = I_{AB}(1 - 1\angle 120^o) = I_{AB}\sqrt{3}\angle -30^o$$

$$I_b = I_{BC} - I_{AB} = I_{BC}(1 - 1\angle 120^o) = I_{BC}\sqrt{3}\angle -30^o$$

$$I_c = I_{CA}\sqrt{3}\angle -30^o$$

Balanced 3-phase systems

Balanced Y-Δ Connection



$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle 120^o) = \mathbf{I}_{AB}\sqrt{3}\angle -30^o$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_{BC}(1 - 1\angle 120^o) = \mathbf{I}_{BC}\sqrt{3}\angle -30^o$$

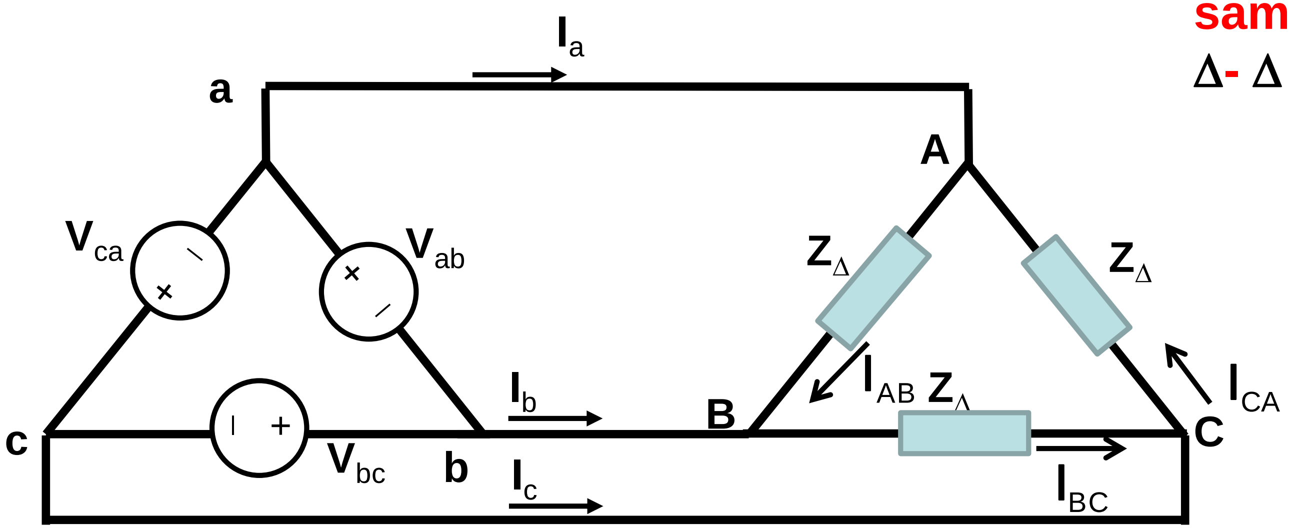
$$\mathbf{I}_c = \mathbf{I}_{CA}\sqrt{3}\angle -30^o$$

$$I_L = \sqrt{3} I_p$$

where $I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$ and $I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$

Phase current LEADS line current by 30°

Balanced 3-phase Δ-Δ



Line-line voltage is the same as phase voltage in Δ- Δ

$$V_{ab} = V_p\angle 0^o$$

$$V_{bc} = V_p\angle -120^o$$

$$V_{cn} = V_p\angle 120^o$$

$$V_{ab} = V_{AB} \qquad I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$V_{bc} = V_{BC} \qquad I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$V_{ca} = V_{CA} \qquad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Phase currents

Using KCL,

$$I_a = I_{AB} - I_{CA}$$
$$= I_{AB}(1 - 1\angle 120^o)$$
$$= I_{AB}\sqrt{3}\angle -30^o$$

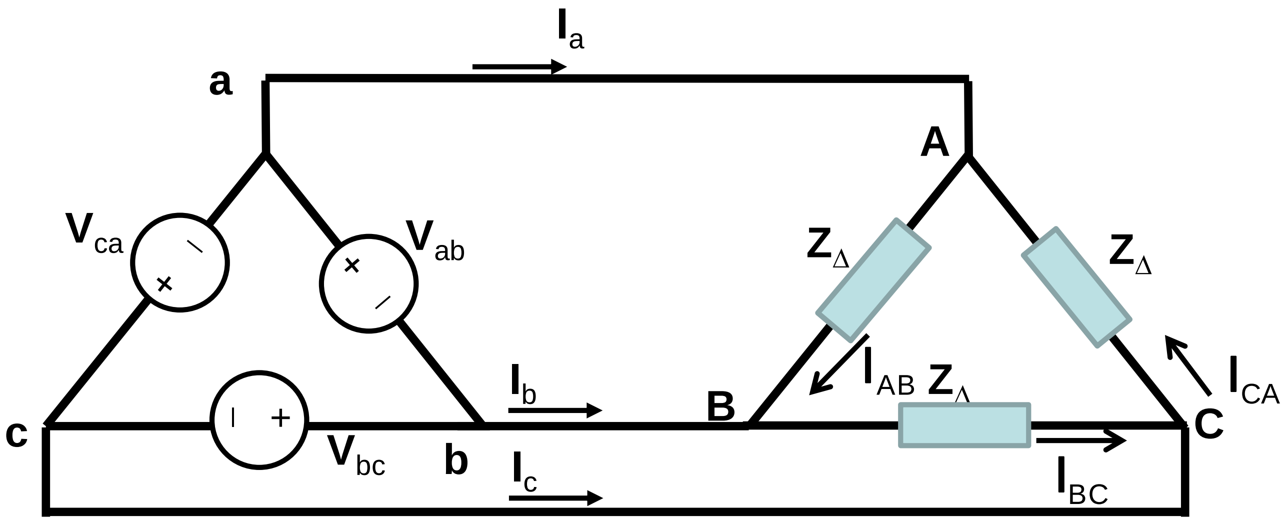
$$I_b = I_{BC} - I_{AB}$$
$$= I_{BC}(1 - 1\angle 120^o)$$
$$= I_{BC}\sqrt{3}\angle -30^o$$

$$I_c = I_{CA}\sqrt{3}\angle -30^o$$

line currents

Balanced 3-phase systems

Balanced Δ-Δ Connection



$$V_{ab} = V_p \angle 0^o$$

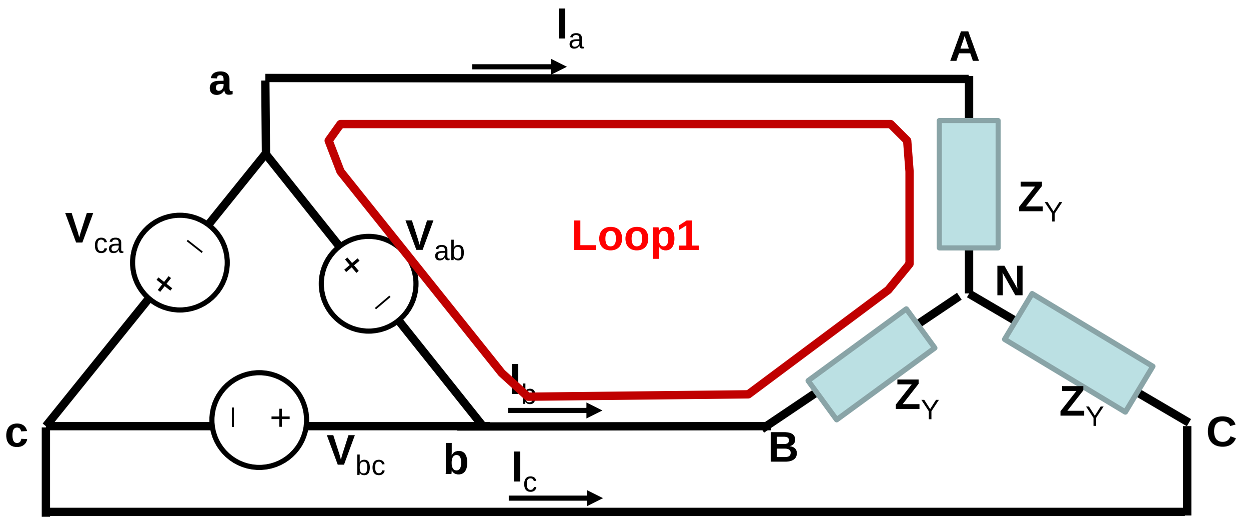
$$V_{bc} = V_p \angle -120^o$$

$$V_{cn} = V_p \angle 120^o$$

Alternatively, by transforming the Δ connections to the equivalent Y connections per phase equivalent circuit analysis can be performed.

Balanced 3-phase systems

Balanced Δ-Y Connection



$$V_{ab} = V_p\angle 0^o$$

$$V_{bc} = V_p\angle -120^o$$

$$V_{ca} = V_p\angle 120^o$$

How to find $\mathbf{I}_a$?

Loop1 $-V_{ab} + Z_Y\mathbf{I}_a - Z_Y\mathbf{I}_b = 0 \qquad \Rightarrow \mathbf{I}_a - \mathbf{I}_b = \dfrac{V_{ab}}{Z_Y}$

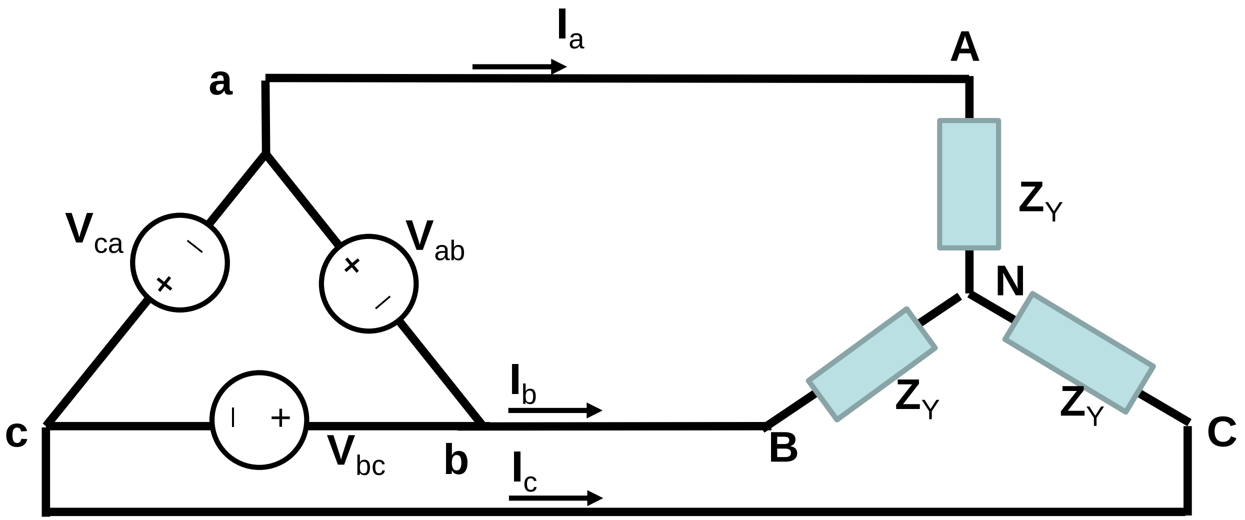
Since circuit is balanced, $\mathbf{I}_b = \mathbf{I}_a\angle\text{-}120^o$ $\qquad \Rightarrow \mathbf{I}_a - \mathbf{I}_b = \mathbf{I}_a(1 - 1\angle(-120^o))$

$$= \mathbf{I}_a\sqrt{3}\angle 30^o$$

Therefore $\quad \mathbf{I}_a = \dfrac{V_p/\sqrt{3}}{Z_Y}\angle -30^o$

Balanced 3-phase systems

Balanced Δ-Y Connection



$$V_{ab} = V_p \angle 0^o$$

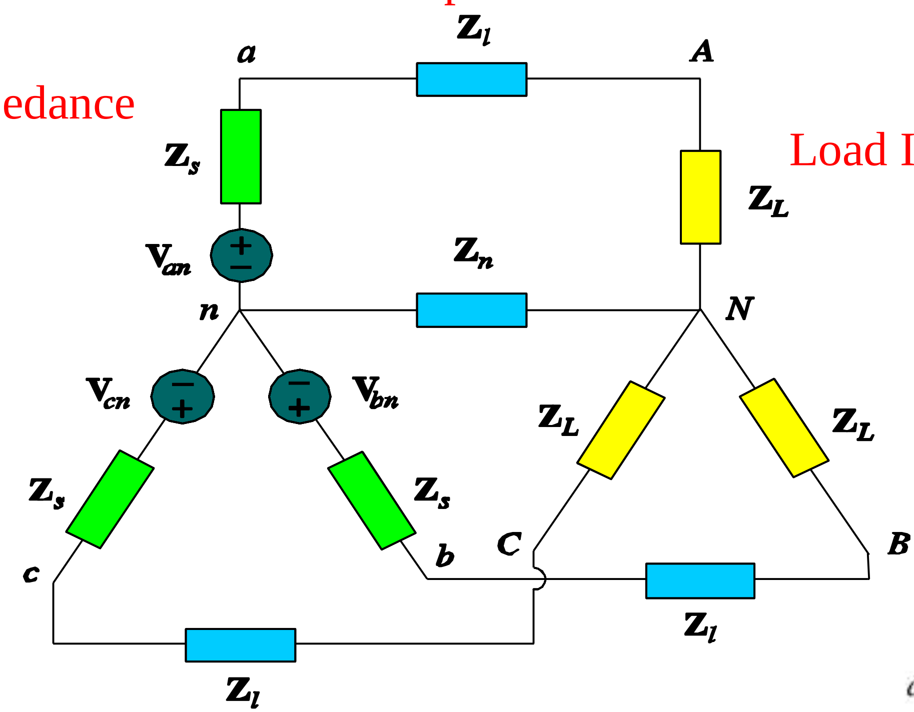
$$V_{bc} = V_p \angle -120^o$$

$$V_{ca} = V_p \angle 120^o$$

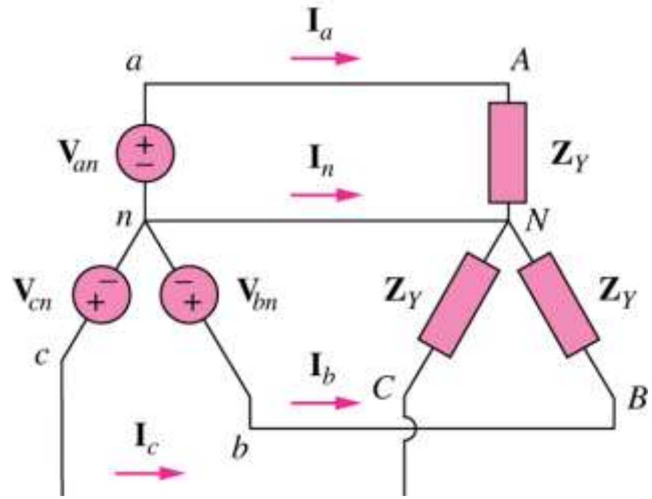
How to find $\mathbf{I}_a$? (Alternative)

Transform the delta source connection to an equivalent Y and then perform the per phase circuit analysis

➢ A balanced Y-Y system, showing the source, line and load impedances.



Equivalent Circuit



Three-phase Circuits

Unbalanced 3-phase systems

Power in 3-phase system

UNBALANCED DELTA-CONNECTED LOAD

The line currents will not be equal nor will they have a 120° phase difference as was the case with balanced loads.

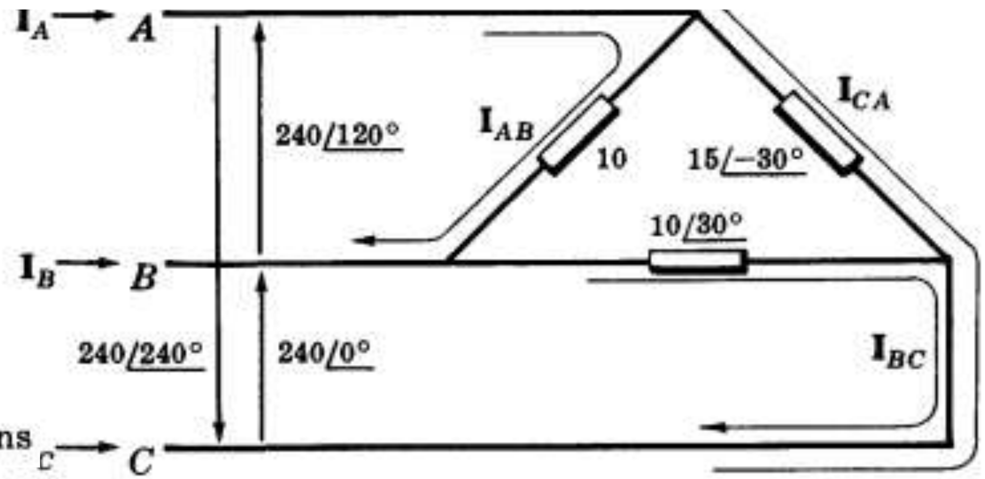
Example 4.

A three-phase, three-wire, 240 volt, *ABC* system has a delta-connected load with $\mathbf{Z}_{AB} = 10\underline{/0^\circ}$, $\mathbf{Z}_{BC} = 10\underline{/30^\circ}$ and $\mathbf{Z}_{CA} = 15\underline{/-30^\circ}$. Obtain the three line currents and draw the phasor diagram.

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{240\underline{/120^\circ}}{10\underline{/0^\circ}} = 24\underline{/120^\circ}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = 24\underline{/-30^\circ}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = 16\underline{/270^\circ}$$



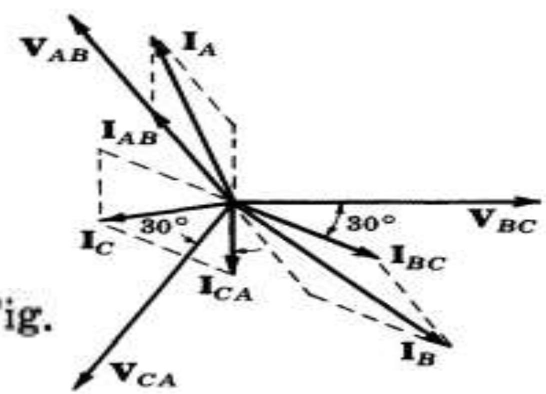
Apply Kirchhoff's current law to the junctions

$$\mathbf{I}_A = \mathbf{I}_{AB} + \mathbf{I}_{AC} = 24\underline{/120^\circ} - 16\underline{/270^\circ} = 38.7\underline{/108.1^\circ}$$

$$\mathbf{I}_B = \mathbf{I}_{BA} + \mathbf{I}_{BC} = -24\underline{/120^\circ} + 24\underline{/-30^\circ} = 46.4\underline{/-45^\circ}$$

$$\mathbf{I}_C = \mathbf{I}_{CA} + \mathbf{I}_{CB} = 16\underline{/270^\circ} - 24\underline{/-30^\circ} = 21.2\underline{/190.9^\circ}$$

The corresponding phasor diagram is shown in Fig.



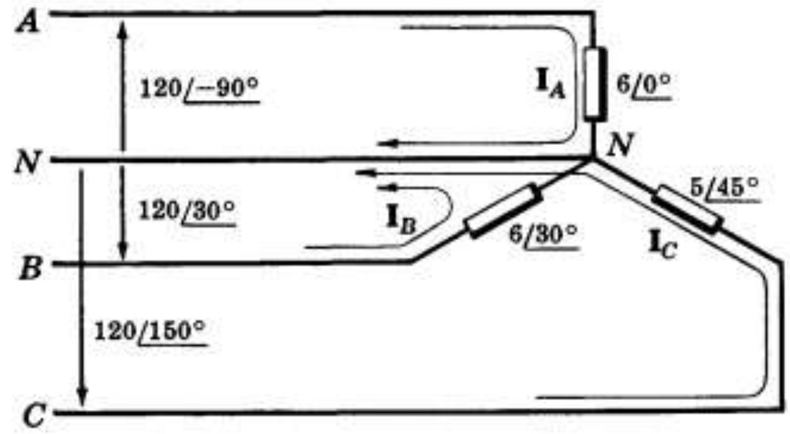
UNBALANCED FOUR-WIRE, WYE-CONNECTED LOAD

- On a four-wire system the neutral conductor will carry a current when the load is unbalanced
- The voltage across each of the load impedances remains fixed with the same magnitude as the line to neutral voltage.
- The line currents are unequal and do not have a 120° phase difference.

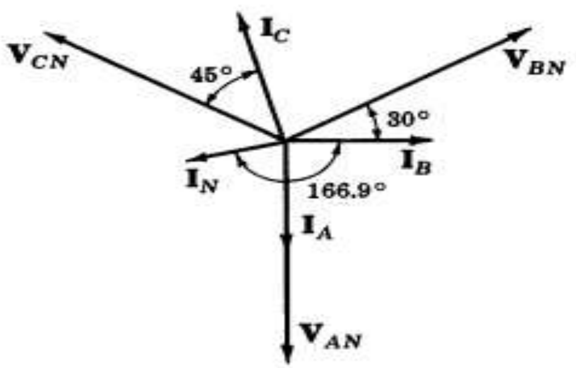
Example 5.

A three-phase, four-wire, 208 volt, *CBA* system has a wye-connected load with $\mathbf{Z}_A = 6\underline{/0^\circ}$, $\mathbf{Z}_B = 6\underline{/30^\circ}$ and $\mathbf{Z}_C = 5\underline{/45^\circ}$. Obtain the three line currents and the neutral current. Draw the phasor diagram.

$$\mathbf{I}_A = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A} = \frac{120\underline{/-90^\circ}}{6\underline{/0^\circ}} = 20\underline{/-90^\circ}$$



$$\mathbf{I}_B = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B} = 20\underline{/0^\circ} \qquad \mathbf{I}_C = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C} = 24\underline{/105^\circ}$$



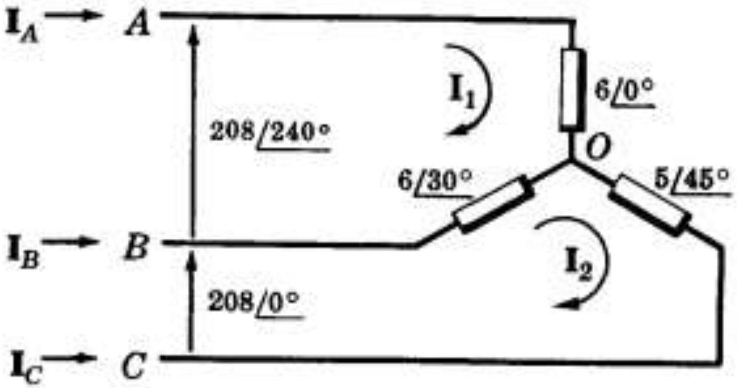
$$\mathbf{I}_N = -(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C) = -(20\underline{/-90^\circ} + 20\underline{/0^\circ} + 24\underline{/105^\circ}) = 14.1\underline{/-166.9^\circ}$$

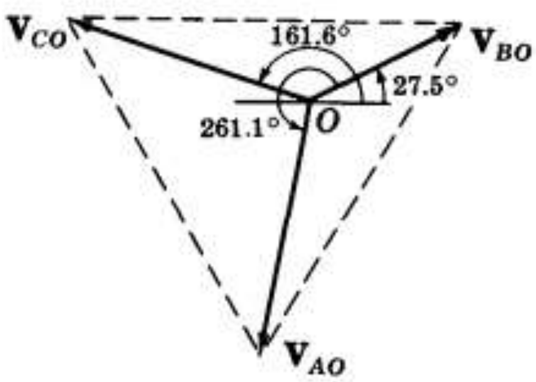
UNBALANCED THREE-WIRE, WYE-CONNECTED LOAD

- The common point of the three load impedances is not at the potential of the neutral and is marked "O" instead of N.
- The voltages across the three impedances can vary considerably from line to neutral magnitude, as shown by the voltage triangle which relates all of the voltages in the circuit.

Example 6.

A three-phase, three-wire, 208 volt, *CBA* system has a wye-connected load with $\mathbf{Z}_A = 6\underline{/0^\circ}$, $\mathbf{Z}_B = 6\underline{/30^\circ}$ and $\mathbf{Z}_C = 5\underline{/45^\circ}$. Obtain the line currents and the phasor voltage across each impedance. Construct the voltage triangle and determine the displacement neutral voltage, $\mathbf{V}_{ON}$.





- Draw the circuit diagram and select mesh currents as shown in Fig.
- Write the corresponding matrix equations (Crammer Rule)

UNBALANCED THREE-WIRE, WYE-CONNECTED LOAD

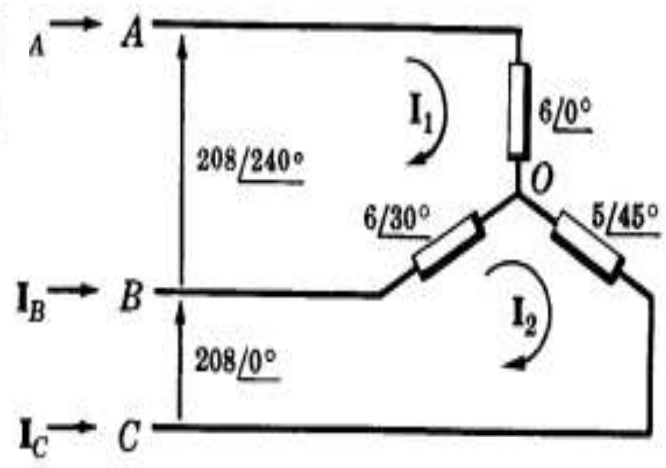
$$\begin{bmatrix} 6\angle 0^\circ + 6\angle 30^\circ & -6\angle 30^\circ \\ -6\angle 30^\circ & 6\angle 30^\circ + 5\angle 45^\circ \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 208\angle 240^\circ \\ 208\angle 0^\circ \end{bmatrix}$$

$$\mathbf{I}_1 = 23.3\angle 261.1^\circ \qquad \mathbf{I}_2 = 26.5\angle -63.4^\circ$$

$$\mathbf{I}_A = \mathbf{I}_1 = 23.3\angle 261.1^\circ$$

$$\mathbf{I}_B = \mathbf{I}_2 - \mathbf{I}_1 = 26.5\angle -63.4^\circ - 23.3\angle 261.1^\circ = 15.45\angle -2.5^\circ$$

$$\mathbf{I}_C = -\mathbf{I}_2 = 26.5\angle 116.6^\circ$$



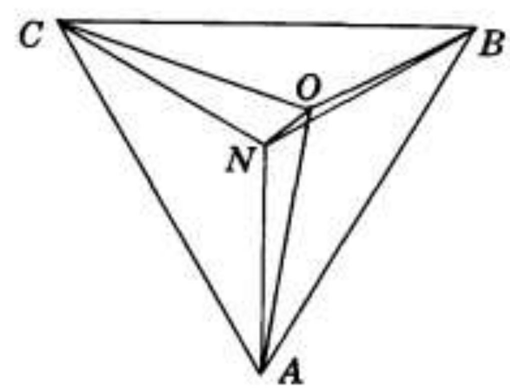
Now the voltages across the three impedances are given by the products of the line currents and the corresponding impedances.

$$\mathbf{V}_{AO} = \mathbf{I}_A\mathbf{Z}_A = 23.3\angle 261.1^\circ\,(6\angle 0^\circ) = 139.8\angle 261.1^\circ$$

$$\mathbf{V}_{BO} = \mathbf{I}_B\mathbf{Z}_B = 15.45\angle -2.5^\circ\,(6\angle 30^\circ) = 92.7\angle 27.5^\circ$$

$$\mathbf{V}_{CO} = \mathbf{I}_C\mathbf{Z}_C = 26.5\angle 116.6^\circ\,(5\angle 45^\circ) = 132.5\angle 161.6^\circ$$

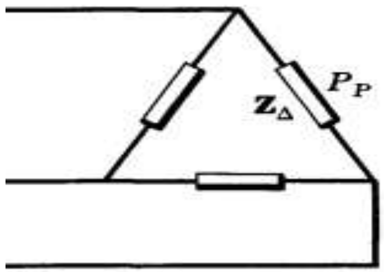
$$\mathbf{V}_{ON} = \mathbf{V}_{OA} + \mathbf{V}_{AN} = -139.8\angle 261.1^\circ + 120\angle -90^\circ = 28.1\angle 39.8^\circ$$

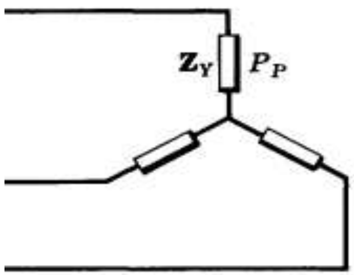


$$\mathbf{V}_{ON} = \frac{\mathbf{V}_{AN}\mathbf{Y}_A + \mathbf{V}_{BN}\mathbf{Y}_B + \mathbf{V}_{CN}\mathbf{Y}_C}{\mathbf{Y}_A + \mathbf{Y}_B + \mathbf{Y}_C}$$

POWER IN BALANCED THREE-PHASE LOADS

➢ Since the phase impedances of balanced wye or delta loads contain equal currents, the phase power is one-third of the total power.





- The voltage across is line voltage
- The current is phase current.
- The angle between V & I is the angle on the impedance.

Phase power $P_P = V_L I_P \cos\theta$

Total power $P_T = 3 V_L I_P \cos\theta$

For a balanced Δ-connected loads:

$$I_L = \sqrt{3}\, I_P$$

$$P_T = \sqrt{3}\, V_L I_L \cos\theta$$

- The voltage across is phase voltage
- The current is line current.
- The angle between V & I is the angle on the impedance.

Phase power $P_P = V_P I_L \cos\theta$

Total power $P_T = 3 V_P I_L \cos\theta$

For a balanced Y-connected loads:

$$V_L = \sqrt{3}\, V_P$$

$$P_T = \sqrt{3}\, V_L I_L \cos\theta$$

POWER IN BALANCED THREE-PHASE LOADS

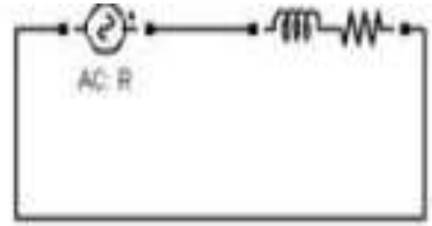
$$P_T = \sqrt{3}\, V_L I_L \cos\theta$$

$$S_T = \sqrt{3}\, V_L I_L$$

$$Q_T = \sqrt{3}\, V_L I_L \sin\theta$$

INSTANTANEOUS THREE-PHASE POWER

➢ Remember: The instantaneous Single-phase power

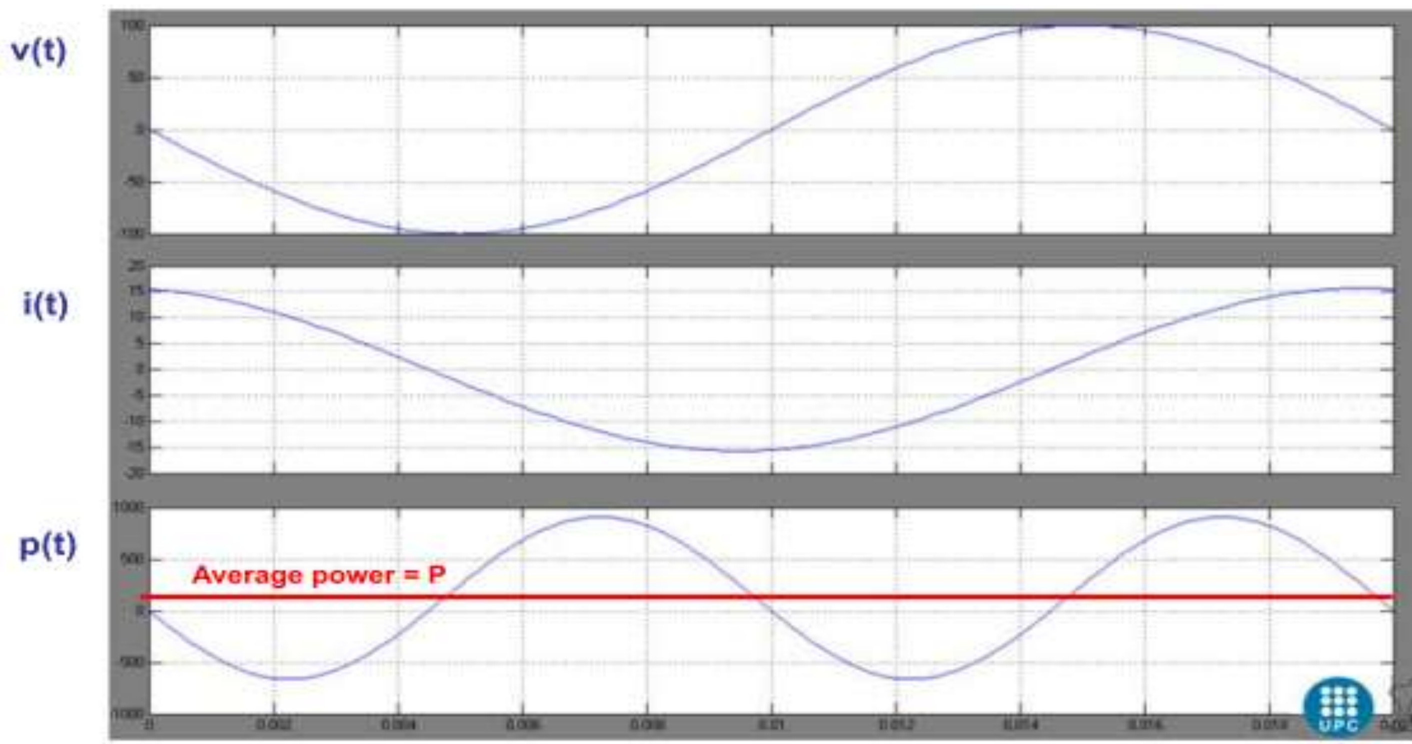


$$p(t) = v(t)\cdot i(t) = V_0\cdot\cos(w\cdot t + \varphi_V)\cdot I_0\cdot\cos(wt + \varphi_I)$$

$$\cos A\cdot\cos B = 0{\cdot}5\cdot[\cos(A+B) + \cos(A-B)]$$

$$p(t) = \underbrace{1/2\cdot V_0\cdot I_0\cdot\cos(\varphi_V - \varphi_I)}_{\text{Constant}} + \underbrace{1/2\cdot V_0\cdot I_0\cdot\cos(2wt + \varphi_V + \varphi_I)}_{\text{Oscillates at twice the mains frequency!}} \text{ watt}$$

Constant **Oscillates at twice the mains frequency!**



INSTANTANEOUS THREE-PHASE POWER

The instantaneous 3-phase power

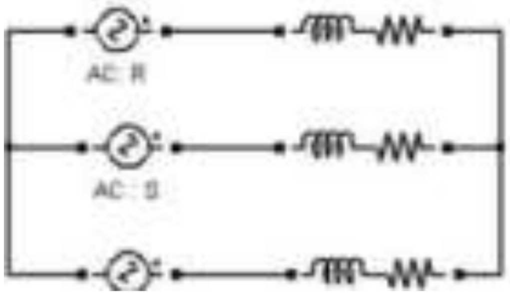
$$p(t) = V_{AN}\, Ia + V_{BN}\, Ib + V_{CN}\, IC$$

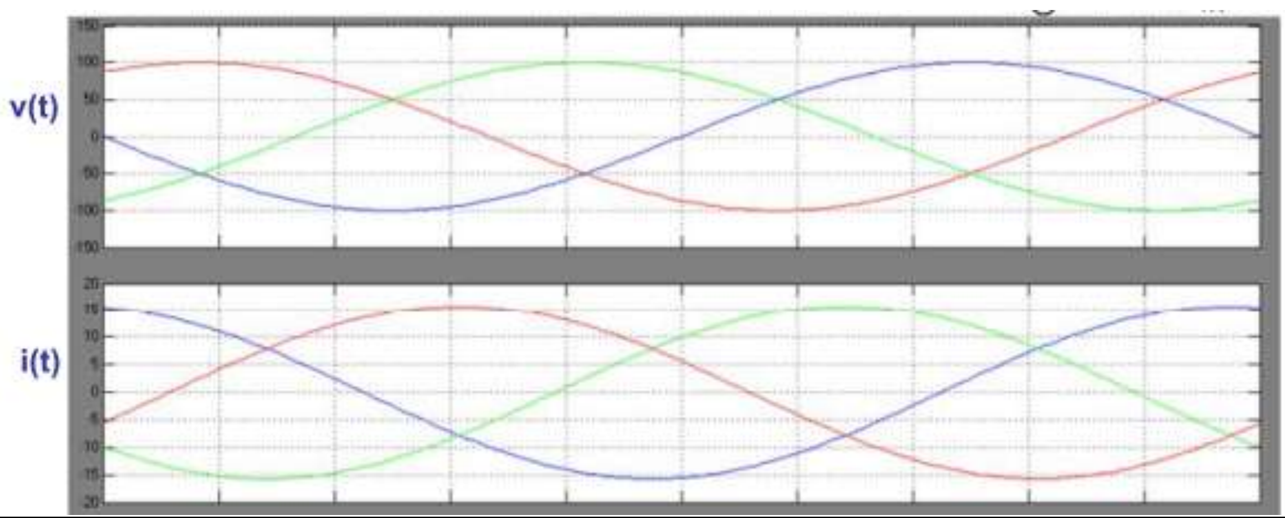
$$= \sqrt{2}V_p \cdot \cos(wt + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt + \varphi_I)$$

$$+ \sqrt{2}V_p \cdot \cos(wt - 120^o + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt - 120^o + \varphi_I)$$

$$+ \sqrt{2}V_p \cdot \cos(wt + 120^o + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt + 120^o + \varphi_I)$$

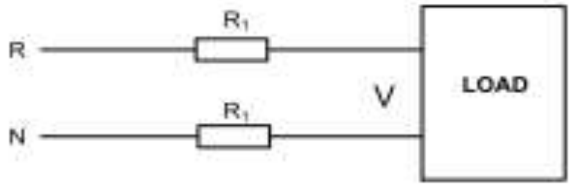
$$= 3/2 \cdot V_p \cdot I_p \cdot \cos(\varphi_V - \varphi_I) = \mathbf{3/2 \cdot V_p \cdot I_p \cdot \cos\varphi = constant!}$$





POWER LOSSES: THREE-PHASE/SINGLE PHASE

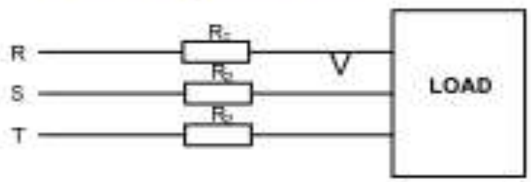
Single-phase line



$$I = \frac{P_{load}}{V \cdot \cos\varphi}$$

$$P_{losses} = 2 \cdot R_1 \cdot I^2 = 2 \cdot R_1 \cdot \frac{P_{load}^2}{V^2 \cdot \cos^2\varphi}$$

Three-phase line



$$I = \frac{P_{load}}{\sqrt{3} \cdot V \cdot \cos\varphi}$$

$$P_{losses} = 3 \cdot R_2 \cdot I^2 = 3 \cdot R_2 \cdot \frac{P_{load}^2}{(\sqrt{3})^2 . V^2 \cdot \cos^2\varphi} = R_2 \cdot \frac{P_{load}^2}{V^2 \cdot \cos^2\varphi}$$

Supposing same losses $2R_1 = R_2 \rightarrow 2\rho\frac{l}{S_{1p}} = \rho\frac{l}{S_{3p}} \rightarrow S_{3p} = \frac{1}{2}S_{1p}$

Single-phase line: 2 conductors of length l and section S_{1p}

Three-phase line: 3 conductors of length l and section $S_{3p} = 1/2S_{1p}$

As a result: $weight_{3p\text{-}cables} = 3/4 weight_{1p\text{-}cables}$

$$\frac{V_{3p}}{V_{1p}} = \frac{3*(l*S_{3p})}{2*(l*S_{1p})} = \frac{3*(S_{1p}/2)}{2*(S_{1p})} = \frac{3}{4}$$